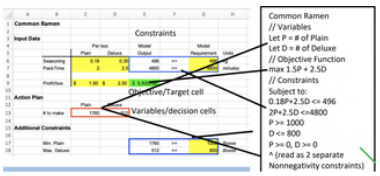
Formulas

slope = -(c1/c2)

- x-intercept (set y=0 and solve)
  - 3x + 4(0) = 8
  - 3x = 8
  - x = 8/3
- y-intercept (set x=0 and solve)
  - 3(0) + 4y = 8
  - 4y = 8




```
Common Ramen
// Variables
Let P = # of Plain
Let D = # of Deluxe
// Objective Function
max 1.5P + 2.5D
// Constraints
Subject to:
0.1BP+2.5D <= 496
2P+2.5D <=4800
P >= 1000
D <= 800
P >= 0, D >= 0
^ (read as 2 separate Nonnegativity constraints)
```

Sensit. Analysis (CONSTRAINT CHANGE?)

1. find slope of constraints, optimal sol.

optimal: 2A + 3B, slope: -2/3
C1: A + B <= 20, slope: -1
C2: A + 2B <= 30, slope: -1/2

2. solve C1 by replacing with variable

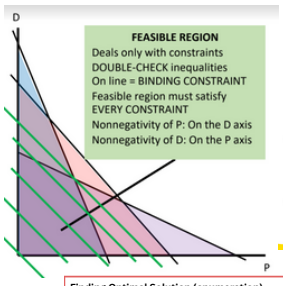
-1 <= -(**C1** / 3) <= -(1 / 2)

left constraint	right constraint
-1 <= -(**C1** / 3)	-(**C1** / 3) <= -(1 / 2)
1 **>=** (C1 / 3)	(C1 / 3) >= (1 / 2)
3 >= C1	C1 >= 3/2

place together, replace C1 with original
(3 / 2) <= 2 <= 3

allowable decrease: 0.5 (2 - (3/2))
allowable increase: 1 (3 - 1)

3. solve C2 by replacing with variable (same)



Mixed References

Relative reference: =G3
Absolute reference: =G3
Mixed reference: =$G3 or =G$3

How Many Variables?

- 2 variables:
  - Can be solved with algebra and graphing.
  - Can be solved in Excel.
  - We can do sensitivity analysis or use the report from Excel.
- 3 or more variables:
  - Cannot be solved graphically.
  - Can be solved in Excel.
  - We still may use algebra to write down the model.

Sensit. Analysis (SHADOW PRICE, RHS values)

shadow price: *marginal benefit of 1 extra item*

1. add 1 to RHS of binding constraint

optimal: min 4H + 8W, slope: -4/8
C1: 0H + 1W >= 5, slope 0
C2: 100H + 140W >= **1401**, slope -100/140

2. find new optimal solution (see finding opt.)
3. **shadow price** = new optimal - old optimal

Finding Optimal Solution (enumeration)

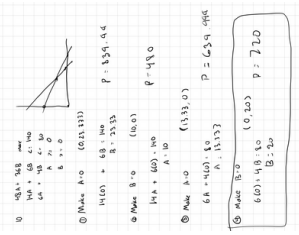
optimal: min 4H + 8W, slope: -4/8
C1: 0H + 1W >= 5, slope 0
C2: 100H + 140W >= 1400, slope -100/140

1. solve system of equations with C1 & C2

0H + 1W >= 5 x140 for all → 0H + 140W >= 700
100H + 140W >= 1400 → 100H + 140W >= 1400
-100H >= 700
H >= 7

solve w
0(7) + 140W >= 700
W = 5
100(7) + 140W >= 1400
W = 5

1. plug values to optimal eq.
4H + 8W → 4(7) + 8(5) = 28 + 40 = 68 (at least 15%)



Shadow price of non-binding constraint: $0.00

Best Equations AG >= 0.15(AG + BG + CG)
=SUMPRODUCT(C10:D10, C15:D15) is same as C10*C15+D10*D15
=SUM(C10:E10) (static percentages)
E13*E21

Objective Coefficient Changes

Change objective co-efficient within allowable range →

- optimal does **not change**
- AND target cell **changes**

Change objective coefficient beyond allowable range →

- Different Binding Constraints
- Different Optimal Solution
- Different Target Cell

Final Value- optimal solution val for that variable

Objective Coefficient– Multiply by final values to get total target cell value

Allowable Increase / Decrease- allowable incr/decr to OBJECTIVE COEFFICIENT before the optimal solution changes

Binding Constraint Changes

Change RHS within allowable range →

- Optimal Solution + Target Cell Change
- Shadow Price * Unit Increased

Change RHS of non-binding constraint →

- Optimal Solution + Target Cell does not Change

If ANY ONE allowable increase/decrease is 0 (TOP PART), the isoprofit line is exactly on one of the binding constraints → multiple optima.

Constraint R.H. Side – The value of the constraint RHS
For blending constraints RHS, most likely 0

Shadow Price – 0 for non-binding constraint, but some binding may have 0. Check if final = RHS. Measured in +1

PROBLEM D.4

Question: Piggie and Gerald's TV Production is considering producing a pilot for a comedy series in the hope of selling it to a major television network. The network may decide to reject the series, but it may also decide to purchase the rights to the series for either one or two years. At this point in time, Piggie and Gerald may either produce the pilot and wait for the network's decision or transfer the rights to the pilot and series to a competitor for $100,000. Piggie and Gerald's decision alternatives and profits (in thousands of dollars) are as follows:

Alternative	State of nature		
	Reject s_1	**1 Year s_2**	**2 Years s_3**
Produce pilot d_1	-100	50	150
Sell to competitor d_2	100	100	100

Simple: What to produce, eat, investments to choose
Blending: How much of each input goes into each output
Scheduling: How many people/resources in each shift
Transportation: How much to ship from location to location
supply and demand do not have to be same.

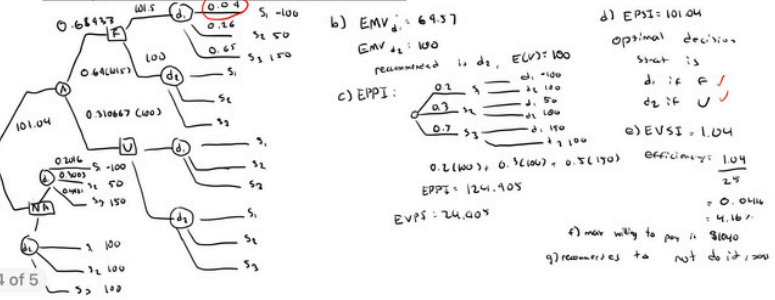
Infeasible = constraints are contradictory, no feasible region
Unbounded = feasible region is infinite, or no corner points

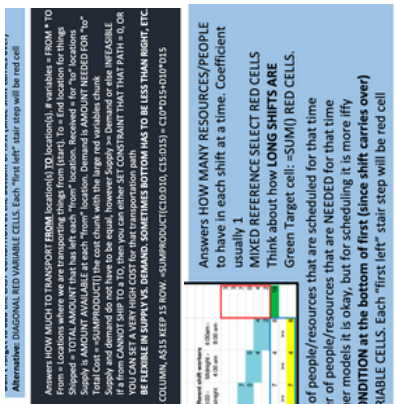
Binding Constraints
Binding constraints: LHS = RHS
Non-Binding constraints: LHS <= RHS
Non-negativity constraint: LHS >= 0

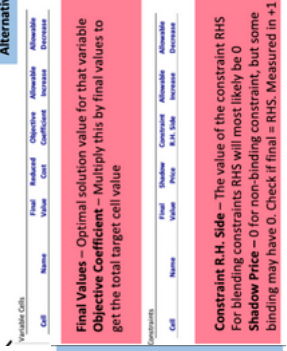
The probabilities for the states of nature are P(S1)=0.2016, P(S2)=0.3003, and P(S3)=0.4981. For a consulting fee of $5000, an agency will review the plans for the comedy series and indicate the overall chance of a favorable network reaction to the series. Assume that the agency review will result in a favorable (F) or an unfavorable (U) review and that the following probabilities are given:

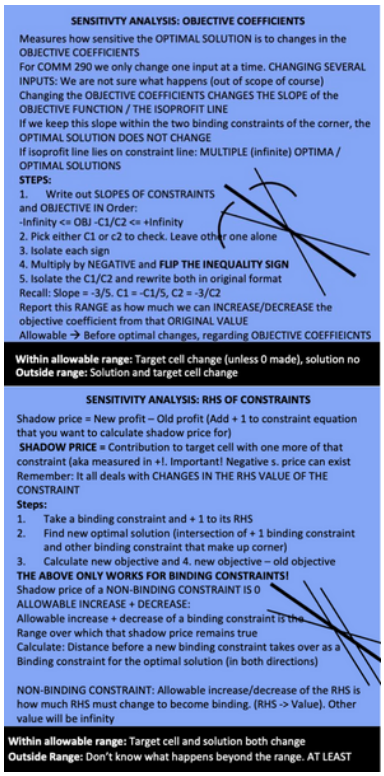
P(F| S1) = 0.3080
P(F| S2) = 0.5974
P(F| S3) = 0.9004

a) Construct a decision tree for this problem.
b) What is the recommended decision if the agency opinion is not used? What is the expected value?
c) What is the expected value of perfect information?
d) What is Piggie and Gerald's optimal decision strategy assuming the agency's information is used?
e) What is the expected value of the agency's information? What is the efficiency of the agency's information?
f) Is the agency's information worth the $5000 fee? What is the maximum that Piggie and Gerald should be willing to pay for the information?
g) What is the recommended decision?

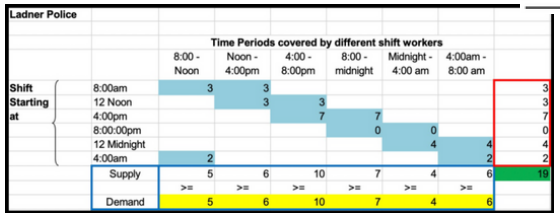


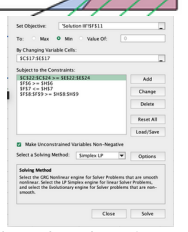


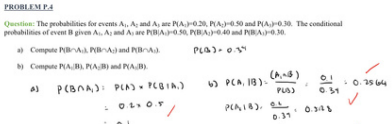




Scheduling LP





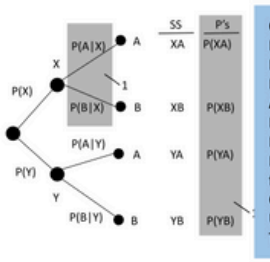


Answers how many resources/people to have in each shift at a time.
Coefficient usually 1 MIXED REFERENCE.
THE RED CELLS: $ the letter.
Variables = # of "shifts"
Think about how LONG each of the shifts are.
Here one person covers 2 shifts.
Green Target cell: =SUM()
RED CELLS. =SUM()

Allowable Increase / Decrease- RANGE for which the SHADOW PRICE HOLDS.
If the allowable increase/decrease is 0, signals demand cannot surpass supply, in transportation LP
Non-binding -> One is how to change RHS to final value, other is infinity (1E+30)
Beyond the RHS allowable range we don't know what happens.

PROBABILITY: Number between 0 and 1
SIMULATION WITH EXCEL:
=RAND() generates the number
=IF(Predicate, True Answer, False Answer)
=COUNTIF(Range, Condition) → Counts NUMBER of
Conditional Formatting: Styling cells based on smth
Notation: P(A u B) = A or B, P(A n B) = A and B
OUTCOME: What happens as result of experiment
List of all possible outcomes: The SAMPLE SPACE
EVENT = A SUBSET of the sample space
Marginal = P(X). Joint = P(A + B)

INDEPENDENT VS. DEPENDENT
I: The outcome of FIRST EVENT DOES NOT AFFECT the outcome of the second
D: First event OUTCOMES HAVE AN EFFECT ON the second effect's probabilities
MUTUAL EXCLUSIVE: If A and B CANNOT HAPPEN SIMULTANEOUSLY. If P(A|B) = 0, they are mut. Ex.
CONDITIONAL PROBABILITY: P(A|B). P(A) given B
P(A|B) = P(AB) / P(B)
BAYES THEOREM: P(A|B) = (P(B|A)*P(B))/P(A)
With Replacement = Inde, Without = Depe



OR: Mut. Ex: = P(A) + P(B).
P(A or B) = P(A) + P(B) – P(AB)
AND: Indep. = P(W)*P(R)
P(A and B) = P(A)*P(B|A). A is first, B is second
Cond=Joint/Mar
Use Joint Prob. Table, find all

AND	A	B	Total
Y	P(AY)	P(BY)	P(Y)
X	P(AX)	P(BX)	P(X)
Total	P(A)	P(B)	1

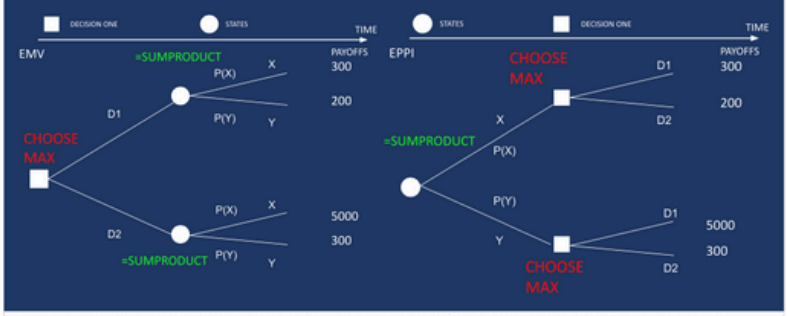
DECISION ANALYSIS: 3 ELEMENTS:
1. Decision (Choices, alternatives, actions)
2. States of Nature (States), no control over
3. Consequences (Outcomes, Payoffs, Results)

PAYOFF MATRIX: Three Decision APPROACHES GIVEN STATES → AND DECISIONS ^ v

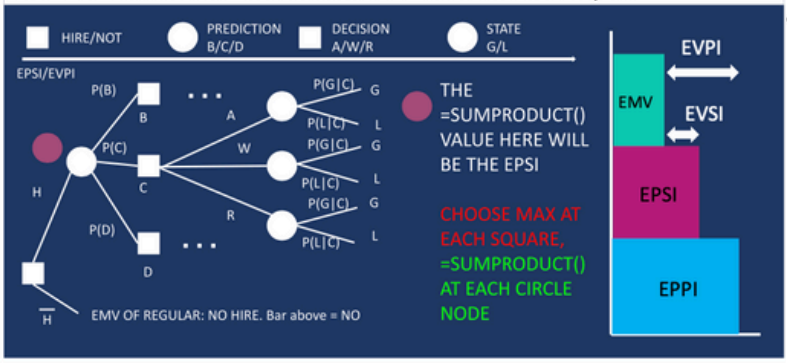
PROFIT	X	Y	Z	CON.	OPT.	REG.
d1	50	80	105	50	105	45 -
d2	70	85	90	70 -	90	60
d3	20	70	150	20	150 -	50

CONSERVATIVE: Looking for the worst possibilities FOR EACH DECISION CHOOSE WORST. Out of those best choose the **HIGHEST OF THE MINIMUMS**
OPTIMISTC: Looking for the best possibilities FOR EACH DECISION CHOOSE BEST. Out of those best choose the **HIGHEST OF THE MAXIMUMS**
REGRET: Looking for the DIFFERENCES in payouts STEPS:
1. Pinpoint the HIGHEST PAYOUT FOR EACH STATE
2. Write out the difference from EACH decision's payout to the best payout.
3. CHOOSE THE HIGHEST REGRET FOR EACH DECISION
4. In the original matrix, choose lowest R. value

← This is given no prob's + outcomes = UNCERTAINTY. With prob's and payoffs = RISK, can find EMV
EMV = EXPECTED MONETARY VALUE = Σ (Payoff)*Probability = Weighted average of each decision



EVPI = EPPI – EMV: THE EVPI IS THE MOST YOU ARE WILLING TO PAY FOR PERFECT INFORMATION
EVSI = EPSI – EMV: MOST WILLING TO PAY FOR SAMPLE INFO. EFFICIENCY: EVSI/EVPI



CONTINUOUS VS. DISCRETE VARIABLES:
DISCRETE: X=0,1,2,3. CONTINUOUS: 0 <= X < 4
P(X=2) = 0.25 P(X=2) = 0
P(X<=2) = 0.75 P(X<=2) = 0.50
P(X<2) = 0.50 P(X<2) = 0.50
RANDOM VARIABLE: Numerical descriptions of the outcome of an experiment. PR. DISTRIBUTION of X:

X	5	6	7	8	9	10
P(X)	.3	.1	.2	.1	.1	.2

EXPECTED VALUE E(X):
Σ X * P(X). IF it's f(x), Σ f(x) * P(X)
With replacement E(X) = without r. E(X)
VARIANCE σ2(X) or VAR(X):
Σ [X – E(X)]^2 * P(X), E(X^2)*[E(X)]^2
DIFFERENCE between each X and E(X) squared, multiplied by probability
STANDARD DEVIATION σ(X):
(sqrt (VAR(X))
UNITS: E(X), ST.DEV = Original, VAR = ^2

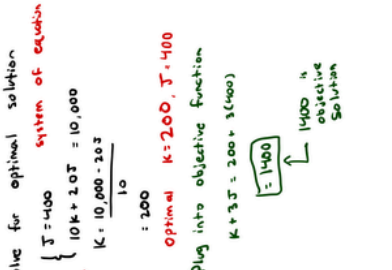
EXCEL: =RANDBETWEEN(LO, HI) produces INTEGER VALUE IN THAT RANGE, INCLUSIVE
OR CAN USE =RAND(), AND IF(=G5<=1/5, "T", "F") are examples of ways to do this with excel
CONSTANT TIMES VARIABLE E(C*X)
E(X) of E(C*X) → E(C*X) = C*E(X)
VAR(X) of VAR(C*X) -> VAR(C*X) = C^2*VAR(X)
ST.DEV(X) of ST.DEV(C*X) -> ST.DEV(C*X) = |C|*ST.DEV(X)

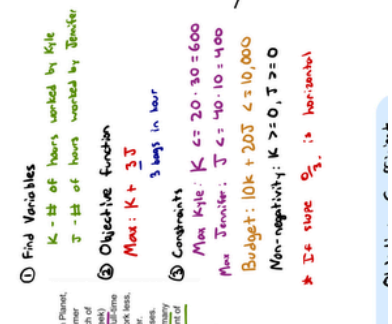
ADDITION OF TWO RANDOM VARIABLES (Given T = A + B), Indep + Dep
INDEPENDENT = P(X and Y) = P(X) * P(Y) holds for all X and Y. No = DEP.
E(T) = E(A) + E(B) no matter independent or dependent
VAR(T) = VAR(A) + VAR(B) if INDEPENDENT, does not hold if dependent

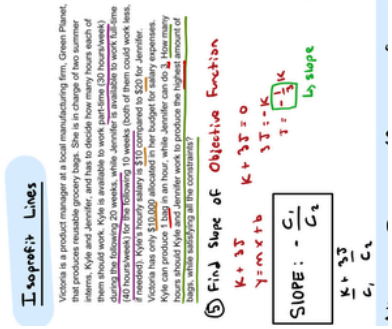
JOINT PROBABILITY DISTRIBUTION BETWEEN 2 RANDOM VARIABLES

AND	A	B	Total
Y	P(AY)	P(BY)	P(Y)
X	P(AX)	P(BX)	P(X)
Total	P(A)	P(B)	1

Recall: E(X – Y) = E(X) + -E(Y)
If relationship between X and Y is NOT addition/subtraction, construct p.tree and use definitions of VAR, ST.DEV, E(X) and calculate manually







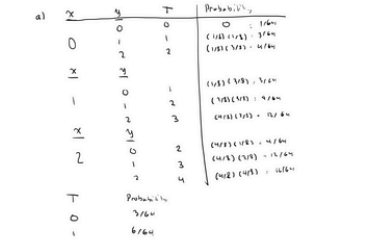
PROBLEM R.3

Question: Let X and Y be independent random variables with common distribution given below:

	Probability
0	1/8
1	3/8
2	1/2

Let T = X + Y.

a) What is the distribution of T?
b) What is the expected value of T?
c) What is the variance of T?



PROBLEM R.5

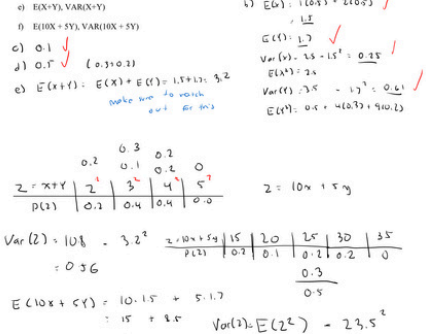
Question: Given the bivariate probability distribution:

		Y			
		1	2	3	
X	1	0.2	0.1	0.2	0.5
	2	0.3	0.2	0	0.5
		0.5	0.3	0.2	

Compute:

a) the marginal distribution of X, the marginal distribution of Y
b) E(X), E(Y), VAR(X), VAR(Y)
c) the probability X=1 and Y=2
d) the probability X=2 and Y≤2
e) E(X+Y), VAR(X+Y)
f) E(10X + 5Y), VAR(10X + 5Y)

c) 0.1
d) 0.5 (0.3+0.2)
e) E(X+Y) = E(X) + E(Y) = 1.5+1.3 = 2.8
make sure to work out for this



PROBLEM P.2

Question: A survey of MBA students obtained the following data on "Students' first reason for submitting an application to the school in which they enrolled in."

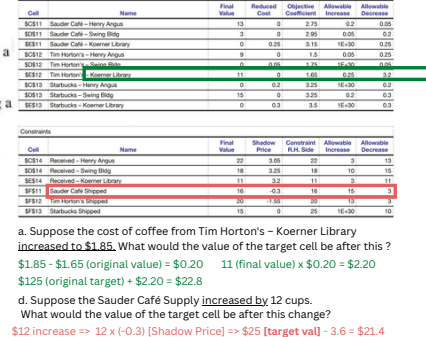
		Reason for Application			
		School Quality	School Cost	Other	Totals
Enrollment	Full Time	421	393	76	890
	Part Time	400	593	46	1039
	Totals	821	986	122	1929

a) If a student goes full time, what is the probability that school quality will be the first reason for choosing a school? P(Q|F) = (421/1929) / (890/1929) = 0.4730

b) If a student goes part time, what is the probability that school quality will be the first reason for choosing a school? P(Q|PT) = (400/1929) / (1039/1929) = 0.3850

c) Let A be the event that a student is full time and let B be the event that student lists school quality as the first reason for applying. Are events A and B independent?

P(A) = 0.4613
P(A|B) = 0.51
no, they are not independent



a. Suppose the cost of coffee from Tim Horton's – Koerner Library increased to $1.85. What would the value of the target cell be after this ?
$1.85 - $1.65 (original value) = $0.20 11 (final value) x $0.20 = $2.20
$125 (original target) + $2.20 = $22.8

d. Suppose the Sauder Café Supply increased by 12 cups.
What would the value of the target cell be after this change?
$12 increase => 12 x (-0.3) [Shadow Price] => $25 [target val] - 3.6 = $21.4