	Production Process	Service Process
Flow Unit	Materials	Customers
Input Rate	Raw material releasing rate	Customer arrival rate
Capacity Rate	Maximum output rate	Maximum service completion rate
Throughput Rate	Finished goods output rate	Customers departure rate (service completion rate)
Flow Time	Time required to turn materials into a product	Time that a customer is being served
Inventory	Amount of work-in-process	Number of customers being served

Throughput rate/Flow Rate is the actual output rate of a process

Throughput rate/Flow Rate is the actual output rate of a process

Utilization (of a resource) = $\frac{\text{Throughput Rate (process)}}{\text{Capacity Rate (resource)}}$ x 100% " $\frac{\text{Actual Output Rate}}{\text{Maximum Output Rate}}$ "

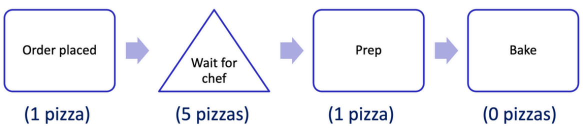
Cycle time is time between outputs = $\frac{1}{\text{Throughput Rate}}$

Implied Utilization = $\frac{\text{Input Rate (process)}}{\text{Capacity Rate (resource)}}$

Capacity Rate = 1/ unit load

Inventory in a buffer counts all flow units in a buffer.
Inventory in an activity counts all flow units performing an activity.
Inventory in the process counts all flow units in the process (this includes all buffers and activities).

Example:



Inventory buildup

Time	Input rate (passengers/15 min slot)	Capacity rate (passengers / 15 min slot)	Excess Demand	Excess Capacity	INVENTORY BUILD-UP in the queue
6:15	7	15	0	8	0
6:30	10	15	0	5	0
6:45	8	15	0	7	0
7:00	12	15	0	3	0
7:15	9	15	0	6	0
7:30	16	15	1	0	1
7:45	14	15	0	1	0
8:00	19	15	4	0	4
8:15	22	15	7	0	11
8:30	17	15	2	0	13
8:45	13	15	0	2	11
9:00	11	15	0	4	7
9:15	12	15	0	3	4
9:30	8	15	0	7	0
9:45	10	15	0	5	0
10:00	7	15	0	8	0
	195	240			3.1875

Empty Buffer (No Queue)

Buffer NOT empty

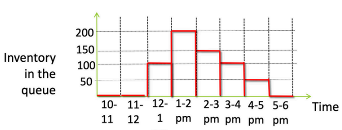
Average Inventory = 3.1875

- A **stable process** is one where

Long-run average *throughput rate* = Long-run average *input rate*

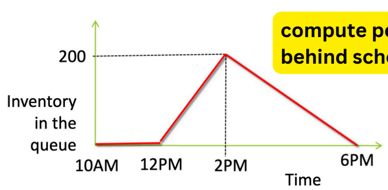
Short-run average input rate can be greater than the short-run average capacity rate

Average Inventory



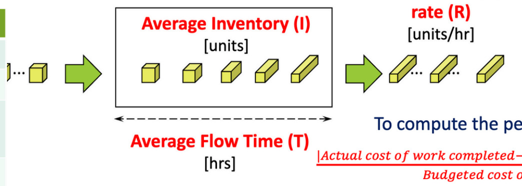
$\text{Average Inventory} = \frac{\text{Area Under the Curve}}{\text{Number of intervals}} = \frac{100 + 200 + 150 + 100 + 50}{8}$

= 75 customers



$\text{Average Inventory} = \frac{\text{Area Under the Curve}}{\text{Length of the Time}} = \frac{600 \text{ Customers}}{8 \text{ hrs}} = 75 \text{ Customers}$

Little's Law



(Average) Inventory = **Little's Law: I = R x T**

Flow Time T = I/R

- Average **throughput rate**, **flow time**, and **inventory** are related
- A manager can influence **any one of these** by **controlling the other two**
- You **cannot independently** choose flow time, throughput and inventory levels

Computer percent project is under/over budget

To compute the percent that a **project** or **activity** is over or under budget…

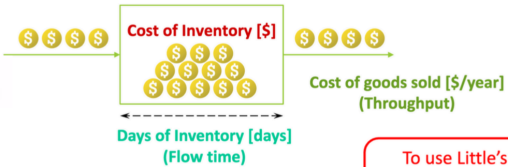
$\frac{|\text{Actual cost of work completed} - \text{Budgeted cost of work completed}|}{\text{Budgeted cost of work completed}}$ x 100% = $\frac{|ACWC - BCWC|}{BCWC}$ x 100%

minutes/unit for unit load
units/hours for capacity rate

high unit load for bottle neck or low capacity rate for bottle neck

Inventory Turnover

- **Days of Inventory :** The number of days inventory stays on the shelf.
- **Inventory turnover:** The number of times inventory is replaced in a year.
- **Cost of goods sold:** Expenses allocated to make the output.



To use Little's Law, we consider average DOI, COI, CGS!

$$\textit{Inventory turnover} = \frac{365}{\textit{Days of Inventory}}$$

- Otto's Auto Parts holds inventory for 2 months, on average.

- Q. What is the inventory turnover in a year?
- Days of Inventory = 60 days
- Inventory Turns = 365 / 60 = 6 (approximately) per year

- Q. If you typically have $10,000 worth of parts, then what is th average cost of goods sold each year?
- Cost of Goods sold = (Turns) x (Cost of Inventory)
- = 6 x $10000 = $60,000

Q. Why are critical paths important?

A. They determine the duration of the process.
A. Any delay in a critical activity will delay the entire process.
A. They allow us to allocate resources among activities to find appropriate trade-offs between duration and cost.

- We could have one resource work on two critical activities, but it might be more cost efficient to hire different resources for each.

Critical Path

Definition: A **critical path** is a longest path through the process flow diagram.

Definition: A **critical activity** is an activity that lies on a critical path.

- Sometimes, activities can be shortened by investing more money.

Definition: A **crash time of an activity** is a new possible (theoretical) flow time.

Definition: The **crash cost** is the price to complete the activity if the flow time is shortened to the crash time.

- We cannot decrease the original flow time beyond the crash time.
- Crash the **"cheapest"** critical activity first (the one with the lowest cost slope).
- When crashing critical activities, pay attention to the fact that the critical path may change as we proceed.

If we want to reduce the process flow time, then we have to reduce the length of all critical paths simultaneously.

Definition: The **cost slope** is the extra cost we pay to shorten the flow time of an activity per time unit.

$$\textbf{Cost slope} = \frac{\textit{Crash Cost} - \textit{Normal Cost}}{\textit{Normal Time} - \textit{Crash Time}}$$

Earned Value Analysis

Definition: Consider the transformation of one flow unit in a process. An **earned value analysis** at a point in time includes

1. The **percentage of scheduled completion** of each activity at this time
2. The **percentage of actual completion** of each activity at this time.
3. The **original budget** for each activity.
4. The **actual cost** spent on an activity up to this point in time.

Definition: The budgeted cost of work scheduled to date for the activity is denoted by BCWS. This equals (% scheduled)*Budget.

*BCWS is how much we ***expected*** to spend based on the ***expected*** amount of completed work.

Definition: The budgeted cost of work completed to date for the activity is denoted by BCWC. This equals (% completed)*Budget.

*BCWC is how much we ***expected*** to spend based on the ***actual*** amount of completed work.

Definition: The actual cost of work completed to date for the activity is denoted by ACWC.

*ACWC is how much we ***actually*** spent based on the ***actual*** amount of completed work.

inventory in process = $I_q + I_s$

compute percent ahead or behind scheule

To compute the percent that an **activity** is ahead or behind schedule…

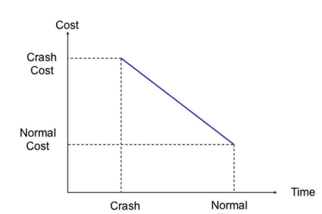
$\frac{|\%\, Scheduled\ (activity) - \%\, Completed\ (activity)|}{\%\, Scheduled\ (activity)}$ x 100%

$= \frac{|BCWS\ (activity) - BCWC\ (activity)|}{BCWS\ (activity)}$ x 100%

To compute the percent that the **project** is ahead or behind schedule…

$\frac{|BCWS\ (Total) - BCWC\ (Total)|}{BCWS\ (Total)}$ x 100%

If BCWS<BCWC, we are ahead of schedule



Multiple Flow Units

Using multiple flow unit analysis, we can look at **average flow units**

Ex. If product A amounts for 25% of our output,
product B amounts for 25% of our output, and
product C amounts for 50% of our output, then....

$\alpha_A = 0.25 \quad \alpha_B = 0.25 \quad \alpha_C = 0.50$

$\alpha_{simple} = \frac{1}{3}$	5
$\alpha_{decorated} = \frac{2}{3}$	15

Product Mix Coefficients

Partner activity

You sell 2 types of cakes – decorated cakes and simple cakes.
You sell twice as many decorated cakes as simple cakes.
When customers ask for a simple cake, the **cashier** removes the cake from the freezer and sells it to the customer.
When customers for a decorated cake, the **designer** removes the cake from the freezer, decorates it, and bring it to the **cashier** to sell.

Removing a cake from the freezer takes 2 minutes,
selling a cake takes 3 minutes,
and decorating takes 10 minutes.

What are the product mix coefficients of a typical order?

$\alpha_d = \frac{2}{3}, \alpha_s = \frac{1}{3}$

What is the unit load for the two different resources (min/cake)?

Cashier expected time = $(\alpha_{simple} \cdot time_{simple}) + (\alpha_{decorated} \times \text{Time for decorated cake})$
$\frac{1}{3} \cdot 5 + (\frac{2}{3} \times 3) = 3.6\overline{6}$

Designer expected time = $(\alpha_{simple} \cdot time_{simple}) + (\alpha_{decorated} + \text{Time for decorated})$
$(\frac{1}{3} \cdot 0) + \frac{2}{3} \cdot 12 = 8$

$3.6\overline{6}, 8$

The designer is bottleneck

Q. What is the process flow time of the product mix?

Process Time Flow: $(\alpha_{simple} \times time_{simple}) + (\alpha_{decorated} \times time_{decorated})$
$\frac{1}{3} \times 5 + \frac{2}{3} \times 15$
$= 11.6\overline{6}$

Scheduling to minimize sum of completion times

The setup:

- Assume n flow units are in the process at time 0.
- Each flow unit has a pre-determined amount of time required to finished the process.

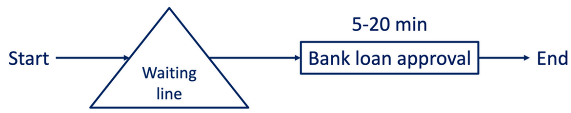
The goal:

Determine in which order the flow units should go through the process so that the sum of completion times is as small as possible.

Shortest processing time algorithm:

1. Among the flow units that are not yet scheduled, choose one with the shortest processing time.
2. Repeat step 1 until all flow units have been scheduled.

Consider a process for approving a bank loan.



Q. How should we schedule the applications in the queue to minimize sum of completion times?

Customer ID	Time to approve loan
1	10 min
2	15 min
3	7 min

Option A

Suppose we schedule 1 then 2 then 3	
Customer ID	Completion time
1	**10**
2	**10+15=25**
3	**10+15+7=32**

10+25+32=67 minutes

Option B

Suppose we schedule 3 then 1 then 2	
Customer ID	Completion time
1	**7**
2	**7+10=17**
3	**7+10+15=32**

7+17+32=56 minutes

Option B is best option

Scheduling with Priorities

The setup:

- Assume n flow units are in the process at time 0.
- Each flow unit has a pre-determined amount of time required to finished the process, and each flow unit has a weight signifying its importance.
- The weighted processing time of a flow unit is (Processing Time/Weight)

The goal:

Determine in which order the flow units should go through the process so that the sum of weighted completion times is as small as possible.

Weighted shortest processing time algorithm:

1. Among the flow units that are not yet scheduled, choose one with the shortest weighted processing time.
2. Repeat step 1 until all flow units have been scheduled.

Customer ID	Time to approve loan in the credit system	Weight	Weighted processing time = Time/Weight
1	6 minutes	4	**1.5**
2	5 minutes	1	**5**
3	8 minutes	2	**4**
4	15 minutes	1	**15**

Customer 1, 3, 2, 4

Scheduling with Due Dates (Moore's Algorithm)

The setup:

- Assume n flow units are in the process at time 0.
- Each flow unit has a pre-determined amount of time required to finished the process, and each flow unit has an assigned deadline.

The goal:

Determine in which order the flow units should go through the process so that the number of units that are late (i.e., miss their deadline) is as few as possible.

Moore's Algorithm:

1. Find the EDD.
2. Among all flow units up to and including the first one that is late, choose the flow unit with the longest processing time and discard it.
3. Repeat Step 2 until every flow unit is either on time or discarded.
4. Schedule all discarded flow units at the end in any order.

How far ahead or behind schedule are we (as a percentage)?

The project is ahead of schedule by $\frac{|42.5-32.5|}{32.5} \times 100\% \approx 30.8\%$

How far over or under budget are we (as a percentage)?

The project is over budget by $\frac{|52.5-42.5|}{42.5} \times 100\% \approx 23.5\%$

Activity	Budget ($)	% Scheduled	% Completed	ACWC ($)	BCWS ($)	BCWC ($)
A	10	50	75	7.5	10 × .5 = 5	10 × .75 = 7.5
B	20	75	50	15	20 × .75 = 15	20 × .5 = 10
C	50	25	50	30	50 × .25 = 12.5	50 × .5 = 25
Total				52.5	32.5	42.5

Suppose you are taking 5 different classes, call them A, B, C, D, and E.
You have different HW assignments in each class.
Each HW assignment has a **due date.**
If you missed the due date, you can **no longer submit the HW.**
The **impact** on your final grade from missing each HW is **the same.**

Class & HW	A-1	A-2	B-1	C-1	C-2	D-1	D-2	E-1	E-2
Days required to finish	5	5	1	1	3	2	2	1	1
Days until due	5	12	5	7	14	6	13	7	14

Q: What order should you do your HW in order to minimize the number of late submissions?

The **earliest due date (EDD) schedule** chooses the earliest deadline.

First Attempt: Use the EDD schedule.

Class & HW	A-1	B-1	D-1	C-1	E-1	A-2	D-2	C-2	E-2
Days required to finish	5	1	2	1	1	5	2	3	1
Days until due	5	5	6	7	7	12	13	14	14
Order	1	2	3	4	5	6	7	8	9
Finish time	5	6	8	9	10	15	17	20	21
Late?	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Second Attempt: Schedule A-1 at the end, and otherwise use EDD.

Class & HW	A-1	B-1	D-1	C-1	E-1	A-2	D-2	C-2	E-2
Days required to finish	5	1	2	1	1	5	2	3	1
Days until due	5	5	6	7	7	12	13	14	14
Order	9	1	2	3	4	5	6	7	8
Finish time	21	1	3	4	5	10	12	15	16
Late?	Yes	No	No	No	No	No	No	Yes	Yes

Third Attempt: Schedule A-1, C-2 at the end, and otherwise use EDD.

Class & HW	A-1	B-1	D-1	C-1	E-1	A-2	D-2	C-2	E-2
Days required to finish	5	1	2	1	1	5	2	3	1
Days until due	5	5	6	7	7	12	13	14	14
Order	9	1	2	3	4	5	6	8	7
Finish time	21	1	3	4	5	10	12	16	13
Late?	Yes	No	No	No	No	No	No	Yes	No

EOQ (Economic Order Quantity) without demand variability

EOQ: Optimal order quantity to minimize inventory cost

D	Annual demand rate
S	Order/setup cost per order
H	Annual holding cost per average unit
L	Lead Time

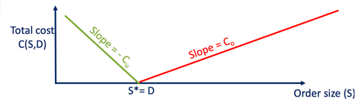
Q: Quantity

Lead time = Reorder point = D * L

$$Q_{OPT} = \sqrt{\frac{2SD}{H}}$$

Goal: Decide how much stock to purchase (S*) to minimize E[C(S,$\mathcal{D}$)].

We plot our (sum of) total overage and underage cost C(S,D) as our order S changes:



$C(S,D) = C_u \times \max\{0, D-S\} + C_o \times \max\{0, S-D\}$

>0 exactly when D-S>0 | >0 exactly when S-D>0

Example:
A doctor's office operates 52 weeks per year, 7 day per week. The office purchases crutches for $6.50 per pair.
The cost of placing an order of crutches from the supplier is $54. The lead time to get an order of crutches is 5 weeks. The average annual cost of storing a pair of crutches is $1.755. The weekly demand for crutches is always 50 pairs per week.

Q: What is the optimal EOQ value?

D	50/wk or 2600/yr
S	$54
H	$1.755 per year

$$Q_{opt} = \sqrt{\frac{2\times54\times2600}{1.755}} = 400 \text{ boxes}$$

Q: What is the corresponding annual total cost TC(Q)?

$$TC(400) = \frac{400}{2}\times1.755 + \frac{2600}{400}\times54 = \$702$$

Q: What should be the reorder point ROP?

ROP = 50 × 5 = 250 boxes

EOQ (Economic Order Quantity) WITH demand variability

$$Q_{OPT} = \sqrt{\frac{2S\times E[\mathcal{D}]}{H}}$$

Triassic Park is known for its dinosaurs. The daily food consumption (in tons of food) C is a normal random variable with mean 100 tons and standard deviation 5 tons. It costs $20 each year to store a ton of food.
It costs the park $400 to place an order for food. The food takes 7 days to reach the park after is has been ordered. Assume 360 days in a year.

Standard Deviation Formula

$$\sigma = \sqrt{\frac{\sum(X-\mu)^2}{n}}$$

boxes of cookies expected to sell

$$E[D] = \sum_d d \cdot \Pr(D = d)$$

d is each possible demand value.

$\Pr(D = d)$ is the probability associated with each demand d.

Computing ROP when demand is normally distributed (EOQ)

$$Q_{OPT} = \sqrt{\frac{2S\times E[\mathcal{D}]}{H}}$$

EOQ (Q_{OPT})	$\sqrt{\frac{2SD}{H}} = 979.8$ units
Average Inventory	$\frac{Q_{OPT}}{2} = 489.9$ units
Average Flow Time (of unit)	$T = \frac{I}{R} = \frac{EOQ/2}{D} =$
Cycle Time (or Reorder Interval)	$\frac{Q_{OPT}}{D} = 0.082$ *years*
Number of orders per year	$\frac{D}{Q_{OPT}} = 12.25$ *times*

Computing ROP when demand is normally distributed (EOQ)

ROP = $E[\mathcal{D}_L]$+SS
or μ_l + SS

$ROP = E[\mathcal{D}_L] + z_{SL}\times\sigma[\mathcal{D}_L]$,

<---- if lead time mentioned, use these formulas

z = NORM.S.INV(CSL) in Excel
(or using a normal table)

SS = z * σ_L

$\sigma_L = \sqrt{k} * \sigma$ **k is the lead time in periods (days, months, years). ex lead time is 1 week, 50 weeks in a year, so k=50**

Q. When should the park order more food if they want to be sure that they run out of food < 1 % of the time during the 7 day waiting period?

Computing ROP when demand is normally distributed

$$E[D_L] = \frac{D}{k} \times \text{Lead Time}$$

$\mu_L = k\mu$

The weekly demand for fancy (pairs of) shoes is normally distributed with an average of 500 (pairs of) shoes and a standard deviation of 50 (pairs of) shoes. The lead time is 2 weeks.

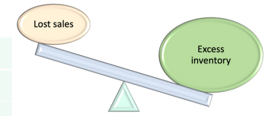
Q. What safety stock is necessary if the store uses a 93% service level?

$\sigma_L = \sqrt{k}\,\sigma = \sqrt{2} * 50 = 70.71$ pairs   SS = $z \cdot \sigma_L$ = 104.65 pairs
z = NORM.S.INV(.93) = 1.48

PK Formula sheet

c	Number of servers
λ	Average arrival rate of units per interval of time (also the throughput rate)
µ	Average service rate (for each server) per interval of time
ρ	Utilization of each server (as well as the resource pool). $\rho = \lambda / (c \times \mu)$ and we need $0 \le \rho < 1$ for the PK-Formula
I_s	The average inventory at the server(s). By Little's Law, $I_s = RT_s = \lambda \times \frac{1}{\mu} = c\rho$
I_q	The average inventory in the queue. (See next page for PK-Formula)
I	The average inventory in the process. $I = I_q + I_s$
T_s	Average flow time at the server. This equals 1/µ.
T_q	Average wait time in the queue. This equals I_q / λ
T	The average flow time to get through the entire process (queue + server). $T = T_s + T_q$

Newsvendor Model



$\mathcal{D}$	(Uncertain) Demand
C_o	Overage cost per unit (cost for ordering one unit above demand)
C_u	Underage cost per unit (cost for not meeting one unit of demand)
S	Stocking quantity

Goal: Decide how much stock to purchase (S) in order to minimize sum of overage and underage costs.

Overage Cost C_o=c-sv
The cost (per unit) of ordering too much is…

Underage Cost C_u=p-c
The cost (per unit) of ordering too few is…

p	Retail price
c	Wholesale cost
sv	Salvage value

We call the optimal value S* or optimum (S)tocking quantity

expected total newsvendor cost

$E[C(S, \mathcal{D})] = C_u \times E[\max\{0, \mathcal{D}-S\}] + C_o \times E[\max\{0, S-\mathcal{D}\}]$

$\mathcal{D}$	The demand during the selling period (This is a random variable)
S*	The optimal order quantity.
p	The retail price per unit (how much we sell each unit for).
c	The wholesale cost per unit (how much we buy each unit for).
sv	The salvage value per unit (how much we salvage each unit for).
C_u	The cost of under ordering, also called the underage cost. This equals p-c.
C_o	The cost of over ordering, also called the overage cost. This equals c-sv.
CR	The critical ratio is $C_u / (C_o+C_u)$. This is the service level that balances C_u and C_o

Q. How much should we order to balance C_u and C_o?

Marginal overage cost		Marginal underage cost	
Probability that $\mathcal{D}\le$S	Pr($\mathcal{D}\le$S)	Probability that S$\le\mathcal{D}$	1-Pr($\mathcal{D}\le$S)
Marginal cost of over stocking	Pr($\mathcal{D}\le$S)×C_o	Marginal cost of under stocking	(1-Pr($\mathcal{D}\le$S))×C_u

The newsvendor solution:
Find the value S* that balances the marginal costs of over- and under-stocking.

$\Pr(\mathcal{D} \le S)\times C_o = (1-\Pr(\mathcal{D} \le S))\times C_u$ **OR** $\Pr(\mathcal{D} \le S) = \frac{C_u}{C_u+C_o}$

S* satisfies $\Pr(\mathcal{D} \le S^*) = \frac{C_u}{C_u+C_o}$

Goal: Decide how much stock to purchase (S*) to minimize total overage and underage costs.

Goal: Decide how much stock to purchase (S*) to minimize E[C(S,$\mathcal{D}$)].

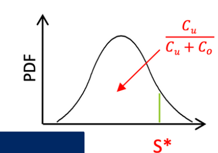
Goal: Decide how much stock to purchase (S*) to balance marginal costs of over-and under-stocking.

All three goals give the same S*!
The newsvendor solution is to order S* satisfying $\Pr(\mathcal{D} \le S^*) = \frac{C_u}{C_u+C_o}$ = z

The value $\frac{C_u}{C_u+C_o}$ is the critical (or newsvendor) ratio (or service level or fractile)

If we have a continuous demand distribution, then…

S* = Any value of S such that $\Pr(\mathcal{D} \le S) = \frac{C_u}{C_u+C_o}$



➢ In the EOQ Model

$ROP = \mu_L + z * \sigma_L$

Probability that demand is less than or equal to the stocking quantity OR **the probability you won't run out of stock** (or your desired service level) known as the CSL or z in the EOQ model

➢ In the NewsVendor Model

S* = E[$\mathcal{D}$] + (z * σ[$\mathcal{D}$])

S* for discrete distribution newsvendor model

If $\mathcal{D}$ follows a discrete distribution, then

S* = smallest S such that Pr($\mathcal{D}\le$S) is at least $\frac{C_u}{C_u+C_o}$.

$\Pr(\mathcal{D} \le S^*) = z = \frac{C_u}{C_u+C_o} = \frac{4}{7}$ z is the critical (or newsvendor) ratio (or service level

Example of Newsvendor model

You need to find the z_(critical ratio). if the ratio is 4/7, find z_(4/7)

You can sell a box of sushi for $7.
Each box costs $3 to make.
At the end of the day,
you throw away any unsold sushi.

Suppose the daily demand $\mathcal{D}$ is normally distributed with E[$\mathcal{D}$]= 50 and σ[$\mathcal{D}$]=5 boxes.

$$\Pr(\mathcal{D} \le S^*) = \frac{C_u}{C_u+C_o} = \frac{4}{7}$$

$z = \frac{4}{7}$ *and you search the z table for* $z_{4/7}$ *or*
$z.57 = 0.1764$

Recall:
C_o = $3
C_u = $4

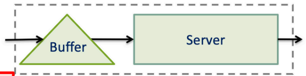
S* = E[$\mathcal{D}$]+$z_{4/7}\times\sigma$[$\mathcal{D}$] ≈ 50+.1764 ×5 = 50.882 boxes

C_o > C_u means higher penalty cost for order too many units

PK Formula (10,13)

(A queue forms in the buffer) Service rate: μ units/ time interval (Average capacity rate)

Arrival rate: λ units/time interval (average input rate)



Throughput rate = λ units/ time interval

Recall we assume stability unless stated otherwise.

exam q

$\lambda = 1 / E[A]$.

$\mu = 1 / E[S] = \frac{1}{T_s}$.

c = num of servers

λ	Average arrival rate (input rate)
$1/\lambda$	Average customer inter-arrival time
μ	Average processing rate (capacity rate)
$1/\mu$	Average processing time
$\rho = \lambda/\mu$ **p = λ/u*c**	Server utilization

PK FORMULA

The system is stable whenever $\lambda \le \mu$, i.e., $\rho \le 1$

$$I_q \cong \frac{\rho^2}{1-\rho} \times \frac{(CA)^2+(CS)^2}{2}$$

"=" for special cases

"≈" in general

$$I_q = \frac{\rho^2}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

For M/M/1 queue, the PK formula is *exact* (=, not ≈)

$L = \frac{\lambda}{\mu - \lambda}$ **For entire process**

$$I_q \cong \frac{\rho^2}{1-\rho} \times \frac{1}{2} = \frac{\lambda^2}{2\mu(\mu-\lambda)}$$

multi-server PK formula (regular one only applies to single)

$$I_q \cong \frac{\rho^{\sqrt{2(c+1)}}}{1-\rho} \times \frac{(C_A)^2+(C_S)^2}{2}$$

$0 \le \rho < 1$ $\rho = \frac{\lambda}{c\mu}$

If it's D/D/1, I_q =0

G/G/1 queue For G/G/x -> p = lambda / u * x

The first "G" refers to the fact that the "**arrivals**" follow a "general" (probability) distribution

The second "G" refers to the fact that the "**service times**" follow a "general" (probability) distribution

The "1" refers to the fact that there is a **single server**

M/M/1 queue

The first "M" indicates that the **inter-arrival times** are **exponentially** distributed (or Markovian)

The second "M" indicates that the **service times** are **exponentially** distributed (or Markovian)

The "1" refers to the fact that there is a **single server**

M/D/1 queue

The first "M" indicates that the **inter-arrival times** are **exponentially** distributed

The second "D" indicates that the **service times** are a **constant** (or Deterministic)

The "1" refers to the fact that there is a **single server**

G/G/c queue

The first "G" refers to the fact that the "**arrivals**" follow a "general" (probability) distribution

The second "G" refers to the fact that the "**service times**" follow a "general" (probability) distribution

The "**c**" refers to the fact that there is are **c many servers**

M/M/c queue, the PK formula is

= 1 in an exponential distribution because the mean and the standard deviation are the same

$$I_q \cong \frac{\rho^{\sqrt{2(c+1)}}}{1-\rho} \times \frac{(C_A)^2+(C_S)^2}{2} = \frac{\rho^{\sqrt{2(c+1)}}}{1-\rho}$$

general **exponential**

M/M/c queue

The first "M" refers to the fact that the "**arrivals**" follow an exponential distribution

The second "M" refers to the fact that the "**service times**" follow an exponential distribution

The "**c**" refers to the fact that there is are **c many servers**

How to deal with variability?

Reduce Variability

- Variability due to "bad information"
- Reduce variability by improving information

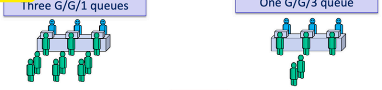
About Input (Demand)	About Process
Better Forecasting	Reduce Process Variability
Better Scheduling	Better Quality

Choose appropriate "Buffer" Build adequate *inventory* and/or Build adequate *capacity*

Reduce impact of variability by "risk pooling"

"Risk pooling" reduces queue length and wait times

Three G/G/1 queues — One G/G/3 queue



$$3 \times \frac{\rho^2}{1-\rho} \times \frac{(C_A)^2+(C_S)^2}{2} > \frac{\rho^{\sqrt{2*4}}}{1-\rho} \times \frac{(C_A)^2+(C_S)^2}{2}$$

if standard devation is mentioned, is G (general probability distribution)

Class 15: EOQ with lead times and random demand

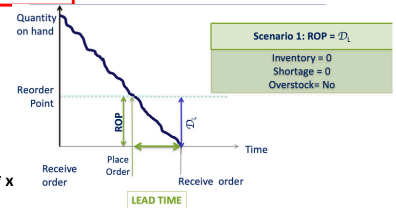
Lead Time (L): Time between placing an order and receiving it.

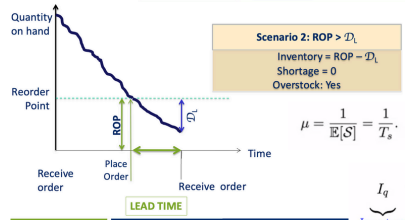
Q: How do we compute the optimal EOQ value Q_{opt} now that the demand is random?

$$Q_{OPT} = \sqrt{\frac{2S \times E[\mathcal{D}]}{H}}$$

Cumulative Distribution Function (CDF) $P(X \le k) = 1 - e^{-\lambda k}$

Probability density function (PDF) $f(k) = \lambda e^{-\lambda k}$

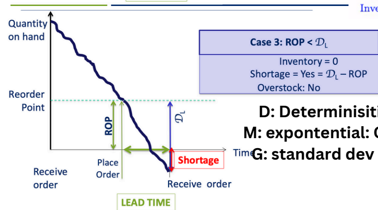




$\mu = \frac{1}{E[S]} = \frac{1}{T_s}$.

$$I_q \approx \frac{\lambda^2}{\mu(\mu-\lambda)} \times \frac{(C_A)^2+(C_S)^2}{2}$$

Inventory — Capacity — Information about variability



D: Deterministic: constant
M: exponential: CA and CV is 1
G: standard dev will be given

Shortage or **stock out** occurs if demand $\mathcal{D}_L$ > ROP.

Cycle Service Level (CSL) or **Service Level** (SL)

- Measure of reliability of the system
- Probability of ***not*** stocking out in a cycle

CSL = Pr($\mathcal{D}_L$ ≤ ROP)

The optimal EOQ quantity is inventory to serve the expected demand during each cycle, i.e., Q is the **cycle stock.**

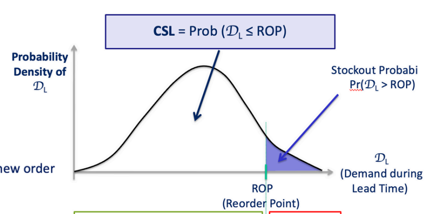
Q. What might happen if the demand rate is uncertain?

A: We might stock out.

- We (the inventory managers) get to choose a service level (this is just the desired probability that we serve demand during lead time).
- -To reach a desired service level, we may need more than just the cycle stock. The extra stock we need to reach the desired service level is the safety stock.

Safety Stock (SS): Inventory used to protect against variable demand.

CSL = Prob ($\mathcal{D}_L$ ≤ ROP)



$\mathcal{D}_L$ is normally distributed $N(\mu_L, \sigma_L^2)$ where $\mu_L = k * \mu$ and $\sigma_L = \sqrt{k} * \sigma$.

μ_L is just another name for E[$\mathcal{D}_L$] — σ_L is just another name for σ[$\mathcal{D}_L$]

use inter-arrival

use avg. process time

Cycle Service Level (CSL) or **Service Level** (SL)

Measure of reliability of the system

Probability of ***not*** stocking out in a cycle

CSL = Pr($\mathcal{D}_L$ ≤ ROP)

- **For an exponentially distributed random variable, the mean and standard deviation are the same.**

Q. How do we determine the ROP?

Recall the Normal Distribution....

Predictable Demand	Unpredictable Demand
D	E[D] is the average expected demand. If E(D) is a random variable that follows a normal distribution, then we say that *X is normally distributed*
σ	σ[D] is the standard deviation of the demand
σ	σ^2 is the variance in the demand

Q: If the lead time is L (in time, e.g., years) and the demand rate is D (in units per time, e.g., units per year), then how many units will be demanded during lead time?

A: D × L

Average waiting time in queue:

$$T_q = \frac{I_q}{\lambda} = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$CV = \frac{Standard\ Deviation}{Mean}$$

Letter 1 / Letter 2 / Number

Indicate how inter-arrival times are distributed — Indicate how service times are distributed — Indicate the number of servers in the system

G = General distribution	Coefficient of variation (CV) equals SD / mean
M = Exponential	CV = 1
D = Deterministic	CV = 0

On Queue Length (Inventory)	On Waiting Time (Flow Time)
$I_q \cong \frac{\rho^2}{1-\rho} \times \frac{(CA)^2+(CS)^2}{2}$	$T_q = \frac{I_q}{\lambda}$ Little's Law

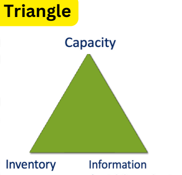
$$Utilization\ \% = \frac{Throughput\ Rate}{Capacity\ Rate} = \frac{Actual\ Output\ Rate}{Maximum\ Output\ Rate} \times 100\%$$

What strategies respond to variability?

- **Inventory** - Let the queue build up.
- **Capacity** – Increase capacity to produce more.
- **Information** - Try doing away with uncertainty by knowing the variability.

The Operations Management (OM) Triangle

- The "Inventory" corner: we have high inventory, low excess capacity, and high variability/low information
- The "Information" corner: High information and low variability, using low excess capacity, and low inventory
- The "Capacity" corner: High excess capacity, using low inventory and information/high variability



The PK-Formula and the OM Triangle

$$I_q \cong \frac{\rho^2}{1-\rho} \times \frac{(CA)^2+(CS)^2}{2}$$

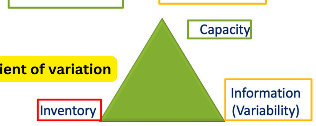
Also denoted as:

$$I_q\ (Inventory) = \frac{\lambda^2}{\mu(\mu-\lambda)} \times \frac{(CA)^2+(CS)^2}{2}\ (Variability)$$

μ= Capacity Rate
λ = Input Rate

Remember: In an exponential distribution The mean and standard deviation will be the same, so CA = 1 and CS = 1

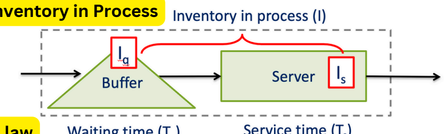
Coefficient of variation



$$Coefficient\ of\ variation\ (CV) = \frac{Standard\ Deviation}{Mean}$$

I_q	Average queue length (this does not include the one customer in service)
ρ	(Long run) Average utilization = Average Throughput / Average Capacity = λ / μ. Note that 0 ≤ ρ < 1.
A	The random variable representing the inter-arrival time between any two consecutive flow units that arrive to the process
E[A]	The average arrival times to a process
$C_A = \frac{\sigma[A]}{E[A]}$	Coefficient of Variation (CV) of inter-arrival times $= \frac{Standard\ Deviation}{Mean} = \frac{\sigma[A]}{E[A]}$
$C_S = \frac{\sigma[S]}{E[S]}$	CV of service times $= \frac{Standard\ Deviation}{Mean} = \frac{\sigma[S]}{E[S]}$

Inventory in Process Inventory in process (I)



Little's law

Waiting time (T_q) — Service time (T_s)

Process flow time (T)

$T_q = I_q / \lambda$ — $I_q = \lambda\, T_q$

$I_s = \lambda\, T_s$

u = 1 / T_s — $I = \lambda\, T$

$T_s = \frac{1}{\mu}$ (utilization rate)

I_q	Average inventory (in queue)
I_s	Average inventory (at the server)
$I = I_q + I_s$	Average inventory (in process)
T_q	Average waiting time (in queue)
T_s	Average service time (at the server)
$T = T_q + T_s$	Average flow time (in process)

EOQ and Inventory Management (14)

Why should a business hold inventory?

Protect against unpredictable variability	Safety Stock
Capitalize on predictable variability	Cycle Stock ("Cycle" refers to between orders)
Possible discounts of economies of scale	Seasonal Inventories
Account for transportation/flow times	Pipeline Inventories

Why should a business not hold inventory?

- Incur carrying costs
- Limited storage
- For inventory with short shelf life, we may waste inventory
- Shifting consumer preferences may make inventory less desirable
- Inventory hides operational problems

How much inventory should we hold for a **particular** type of item?

Trade-off 1: Ordering costs (Economies of Scale) vs Holding costs

Trade-off 2: Shortage costs vs Cost of wasted inventory

Ordering/Setup costs (Fixed costs)	=	Fixed transportation costs Order processing costs
Holding costs (Carrying costs)	=	Costs for storage, insurance, work, tied up working capital
Shortage Costs (Opportunity Costs)	=	Lost sales, etc.

The EOQ Model

Economic Order Quantity (EOQ) balances ordering and holding costs.

Goal: Determine the quantity (Q) to order in order to balance ordering and holding costs.

Goal: The EOQ model determines how much quantity (Q) to order in order to minimize total annual cost.

Notation	
D	Annual Demand Rate
Q	Lot or batch size (the order size)
S	Set-up cost per lot/batch, or average cost of processing/placing an order
C	Unit cost
H	Annual holding/storage cost per unit of average inventory
i	Percent carrying cost (e.g., "interest" rate)

Usually, H = iC.

$$Average\ (cycle)\ inventory = \frac{Area\ under\ curve}{T} = \frac{Q}{2}$$

How to estimate costs

Ordering/setup costs (fixed costs): S

- Estimate costs incurred during the placement of each new order
- Do not count "sunk" costs and fixed overhead

Unit Cost: C

- Estimate variable cost incurred in the production of each additional unit

Holding/carrying costs: H or i

- Estimate opportunity cost of working capital
- Estimate cost of storage, handling, etc.

How many times per year would you place orders per year, i.e., ***frequency*** of ordering? $\frac{D}{Q_{OPT}}$

What is the time in years (called cycle time) between successive order? $\frac{Q_{OPT}}{D}$

The optimal Economic Order Quantity (EOQ) is

$$Q_{OPT} = \sqrt{\frac{2SD}{H}}$$

D	Annual Demand
Q	Lot or batch size (Quantity of replenishment order)
S	Order/setup cost per order
H	Average annual holding/storage cost per unit of inventory

Q. What happens to the cost and optimal quantity as the parameters change?

As	"Cost"	EOQ	Cycle Time
S increases	increases	increases	decreases
H increases	increases	decreases	increases
D increases	increases	increases	increases

$$Q_{OPT} = \sqrt{\frac{2SD}{H}}$$

$$TC(Q) = \frac{Q}{2}H + \frac{D}{Q}S$$

$$Cycle\ Time = \frac{D}{Q_{OPT}}$$

What might affect inventory in the buffer?

Utilization of the server ρ (higher ρ correlates with higher I_q).

The capacity rate μ of the server (higher μ correlates with lower I_q).

Variability in the arrivals and the server.

Number of orders per year	D/Q
Average (cycle) inventory	Q/2 **if no safety stock otherwise Q/2 + SS**

Annual Setup Cost	(D/Q) * S
Annual Holding Cost	(Q/2) * H **or Total Cost / 2**
Annual Total Cost	Annual Setup Cost + Annual Holding Cost

$$Total\ Annual\ Cost = TC(Q) = \frac{Q}{2}H + \frac{D}{Q}S = Annual\ holding\ Cost + Annual\ setup\ costs$$

Class 18: Forecasting (SMA, MAE)

Forecasting is a statistical estimate of future demand, that can be used to plan current activities

Forecasts are often based on past sales, while considering issues like seasonality, trends in demand, etc

Finance and Accounting: Forecasts provide the basis for budgetary planning and cost control.

Marketing: Relies on sales forecasting to plan new products and promotions.

Production and Operations: Use forecasts to make decisions involving capacity planning, process selection and inventory control.

Strategic Planning: Forecasting is one of the bases for corporate long-run planning.

Qualitative Forecasting methods

Executive Judgment
Based on experience and history

Market Research
Surveys, interviews, etc.

Panel Consensus
Meetings of executives, salespeople and customers

Delphi Method

1. Choose experts to participate representing a variety of points of view
2. Obtain forecasts (and reasoning) from all participants
3. Summarize the results and redistribute them to the participants along with appropriate new questions
4. Summarize again, refining forecasts and conditions, and again develop new questions to distribute to all participants
5. Repeat the previous step as necessary and distribute the final results.

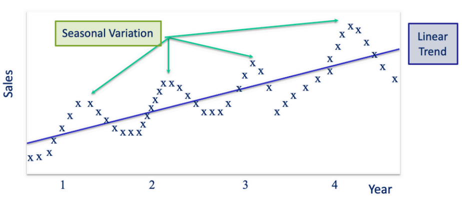
Quantitative Forecasting Methods

Time Series Analysis

Time series forecasting models try to predict future based on past data

Some common approaches
- Moving averages
- Exponential smoothing

A Typical Time-Series of Past Demands



Forecasting notation:

A_t The actual demand at time t.

F_t The forecasted demand at time t.

In COMM 204, we look at different methods of forecasting F_t:
1. Simple moving averages
2. Weighted moving averages
3. Exponential smoothing

For each method, we measure error using the mean absolute error.

Q. How can we quantify a linear trend?

Definition: The **n-period simple moving average (SMA)** at period t is

$$F_t = \frac{A_{t-1} + A_{t-2} + \dots + A_{t-n}}{n}$$

This is one of the simplest forecasts.
This average can remove variability not associated to trends.
n is the window length, which we can choose.

Simple Moving Average: Example

Week	Demand	3-Week SMA	6-Week SMA
1	650	N/A	N/A
2	678	N/A	N/A
3	720	N/A	N/A
4	785	682.67	N/A
5	859	727.67	N/A
6	920	788.00	N/A
7	850	854.67	768.67
8	758	876.33	802.00
9	892	842.67	815.33
10	920	833.33	844
11	789	856.67	866.5
12	844	867	854.83

$F_4 = (650+678+720)/3 \approx 682.67$ $F_7 = (650+678+720+785+859+920)/6 \approx 768.67$

What is a good forecast?

Definition: The **forecast error at time** t is $F_t - A_t$.

Definition: The **mean absolute error (MAE)** at time T is

$$MAE = \frac{\sum_{t=1}^{T} |F_t - A_t|}{T}$$

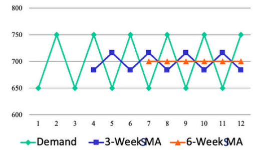
The average is only taken over time periods where we have F_t and A_t.

Example.

Month	Demand	Forecast	Forecast Error
1	220		
2	250	255	5
3	210	205	-5
4	300	320	20
5	325	315	-10

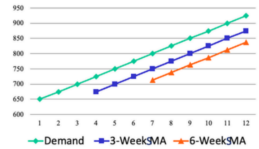
MAE = |5+5+20+10|/4 = 10

Q. Which forecast has the lower MAE?



The 6-week SMA has a lower MAE

No long term trend.
The 6-week SMA does not `fluctuate' as much as the 3-week SMA.



The 3-week SMA has a lower MAE

Long term trend.
The 6-week SMA has a longer lag behind actual demand than 3-week SMA .

Q. How many periods should be used?

Advantages of More Periods	Disadvantages of More Periods
➢ More data points can give a better estimate in the sense that the effect of randomness is reduced by averaging together a number of observations ➢ When there is no trend in the data, using more observations results in a forecast with lower error	➢ A large number of observations causes the moving average to respond slowly to permanent changes ➢ When there is a trend in the data, using more observations results in a forecast with high error

Class 19: Exponential Smoothing and Weighted Moving Average

Definition: Let $w_1, \dots, w_n$ be nonnegative and satisfy $w_1 + \dots + w_n = 1$.
The **n-period weighted moving average (WMA)** at period t is

$$F_t = w_1 \cdot A_{t-1} + w_2 \cdot A_{t-2} + \dots + w_n \cdot A_{t-n}$$

Be careful! Weight w_1 is applied to the most recent observed demand.

Often, greater weight is placed on more recent observations.

Weighted Moving Average: Example

Assume that

Week 3- $w_1 = .50$
Week 2 - $w_2 = .35$
Week 1 - $w_3 = .15$

Q: What is the 3-week WMA for Week 4?

A=696.2 B=675 C=690.8
D=694.80

A	B	C
Week	Demand	3-Week WMA
1	650	N/A
2	678	N/A
3	720	N/A
4	785	????
5	859	
6	920	
7	850	
8	758	
9	892	
10	920	
11	789	
12	844	

Weighted Moving Average: Example

Assume that

$w_1 = .50$
$w_2 = .35$
$w_3 = .15$

Week	Demand	3-Week WMA
1	650	N/A
2	678	N/A
3	720	N/A
4	785	694.80
5	859	746.20
6	920	812.25
7	850	878.40
8	758	875.85
9	892	814.50
10	920	838.8
11	789	885.9
12	844	850.3

$F_4 = (.50 \cdot 720) + (.35 \cdot 678) + (.15 \cdot 650) = 694.80$

Q. How can we introduce `importance' to different measurements?

Weighted moving averages only consider n time periods.
If we want to consider all time periods, then we can perform exponential smoothing.

If α (alpha) is small (i.e., close to 0), more weight is given to observations from the more distant past. **If α is large (i.e., close to 1), more weight is given to the more recent observations.**

Definition: Let α be in (0,1). We call α the smoothing factor. Let F_1 be an initial value. The **exponential smoothing (ES)** at period t ≥ 2 is

$$F_t = F_{t-1} + \alpha \cdot (A_{t-1} - F_{t-1}) = \alpha \cdot A_{t-1} + (1-\alpha) \cdot F_{t-1}$$

You have to determine the forecast error for both the 3 week WMA, and the ES MAE

Exponential Smoothing

$$F_t = F_{t-1} + \alpha \cdot (A_{t-1} - F_{t-1}) = \alpha \cdot A_{t-1} + (1-\alpha) \cdot F_{t-1}$$

$$\begin{aligned} F_t &= \alpha \cdot A_{t-1} + (1-\alpha) \cdot [\alpha \cdot A_{t-2} + (1-\alpha) \cdot F_{t-2}] \\ &= \alpha \cdot A_{t-1} + (1-\alpha) \cdot \alpha \cdot A_{t-2} + (1-\alpha)^2 \cdot [\alpha \cdot A_{t-3} + (1-\alpha) \cdot F_{t-3}] \\ &= \dots \\ &= \alpha \cdot A_{t-1} + (1-\alpha) \cdot \alpha \cdot A_{t-2} + \dots + (1-\alpha)^{t-2} \cdot \alpha \cdot A_1 + (1-\alpha)^{t-1} \cdot F_1 \end{aligned}$$

This is why it is called *exponential* smoothing.

Larger values of α give more weight to the recent observed data A_{t-1}.
Smaller values of α give more weight to the recent forecast F_{t-1}.

Exponential Smoothing: Example **Ex.1**

Assume that

$F_1 = 500$

$F_2 = (1-0.1) \cdot F_1 + 0.1 \cdot A_1$
$= 0.9 \cdot 500 + 0.1 \cdot 650 = 515$

$F_2 = (1-0.9) \cdot F_1 + 0.9 \cdot A_1$
$= 0.1 \cdot 500 + 0.9 \cdot 650 = 635$

Week	Demand	ES α = .1	ES α = .9
1	650	500	500
2	678	515	635
3	720	531.30	673.70
4	785	550.17	715.37
5	859	573.65	778.04
6	920	602.19	850.90
7	850	633.97	913.09
8	758	655.57	856.31
9	892	665.81	767.83
10	920	688.43	879.58
11	789	711.59	915.96
12	844	719.33	801.70

Q: How do we pick up long term trends and seasonal patterns?

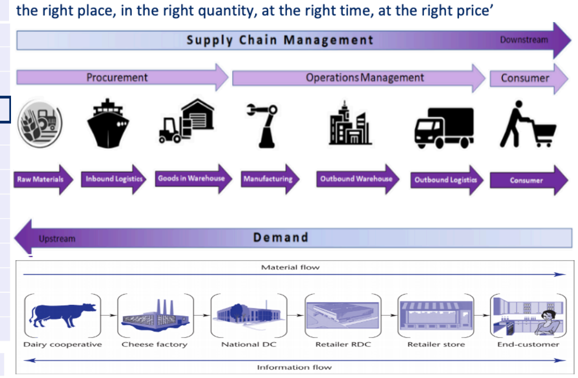
Consider exponential smoothing.

Larger α closely follow recent values of A_t – Seasonal Patterns

A lower α smooths over older values of A_t – Long term trends

Class 20: Supply Chain Management and Supply Chain Design

The supply chain is defined as 'the processes involved in getting the right product, to the right place, in the right quantity, at the right time, at the right price'



➢ The definitions tend to follow one of 3 perspectives:

1. **Network** – organisation of customers (downstream) and suppliers (upstream) who are part of the supply chain; this also includes the suppliers of the raw materials needed for production and final consumer of the products and services.
2. **Flow** – movement of materials, information and cash between suppliers, manufacturers, logistics providers and customers.
3. **Process** – articulation of the core processes involved in the supply chain – planning, purchasing, manufacturing and logistics. More recently this has included the reverse flow of materials at end of life.

The supply chain is actually a **complex supply network of interconnected operations between buyers, suppliers and customers.**

- **The Supply Network is defined as** "the network of organizations that are involved, through upstream and downstream linkages, in the different processes and activities that produce value in the form of products and services delivered to the ultimate consumer (Christopher, 1992).

The Newsvendor Model – how do we balance overage and underage costs?

We place one order. This is different than the EOQ model.

If $\mathcal{D}$ follows a continuous distribution, then

$$S^* = S \text{ such that } \Pr(\mathcal{D} \leq S) = \frac{C_u}{C_u + C_o}$$

If $\mathcal{D}$ follows a discrete distribution, then

$$S^* = \text{smallest } S \text{ such that } \Pr(\mathcal{D} \leq S) \text{ is at least } \frac{C_u}{C_u + C_o}.$$

Suppose $C_u = 5$ and $C_o = 2$.

Assume that the demand follows a discrete distribution:

d	$\Pr(\mathcal{D}=d)$
1	0.15
2	0.25
3	0.35
4	0.25

Q. What is S*?

$\frac{C_u}{C_u + C_o} = 5/7 = .71$

A. S* = 3

How to pick up long term trends and seasonal patterns

Consider exponential smoothing.

Larger α closely follow recent values of A_t – Seasonal Patterns

A lower α smooths over older values of A_t – Long term trends

THE EOQ MODEL

Goal: Determine the quantity (Q) to order in order to `balance' ordering and holding costs.

Notation	
D	Annual Demand Rate
Q	Lot or batch size (the order size)
S	Set-up cost per lot/batch, or average cost of processing/placing an order
C	Unit cost
H	Annual holding/storage cost per unit of average inventory
i	Percent carrying cost (e.g., "interest" rate)

Usually, H = iC.

Flow Chart/Process Flow Diagram: activities, buffers, decision points in proc
Flow Unit: item that flows through a process. May transform (start as person, become order, become set of ingredients, then final product).

Flow time of an activity: Shortest amount of time in a process needed for any flow unit to complete an activity.
Flow time of a process: Shortest amount of time in a process needed for any flow unit to complete the entire process.

Unit load of a resource: time a resource needs to be used to complete one flow unit in the process. This equals the sum of activity times of activity times of activities using this resource.
Resources are "computer, worker 1, etc". Bottlenecks are ONLY resources.

Capacity rate of a resource: Max number of units that a resource can work on in a given time.

Bottleneck resource: resource with LOWEST CAPACITY RATE.
Resource Pool: collection of resources that perform identical activities.

Capacity Rate of the process: The max output rate in a stable state.

Global Production Networks Configuration

CMP

Demand is normally distributed

$$\mathbb{E}[\mathcal{D}_L] = L \times \mu \quad \text{and} \quad \sigma[\mathcal{D}_L] = \sqrt{L} \times \sigma.$$

$$\text{CSL} = \Pr\left(\mathcal{Z} \leq \frac{\text{ROP} - \mathbb{E}[\mathcal{D}_L]}{\sigma[\mathcal{D}_L]}\right).$$

CSL → Z ≤ (ROP − E(D_L)) / σ(D_L)

$$\text{SS} = \text{ROP} - \mathbb{E}[\mathcal{D}_L] = (\mu \times L + z_{\text{CSL}} \times \sqrt{L} \times \sigma) - (\mu \times L) = z_{\text{CSL}} \times \sqrt{L} \times \sigma.$$

Activity	Original Time	Original Cost	Crash Time	Crash Cost
A	15 days	\$50	–	–
B	90 days	\$10	80 days	\$70
C	80 days	\$10	65 days	\$150
D	80 days	\$10	70 days	\$40
E	65 days	\$10	55 days	\$20
F	50 days	\$20	40 days	\$80

(a) What is the critical path(s) and the current duration of the project? (2 marks)

There are three paths: ABEF, ABDF, ACDF, with lengths 220 days, 235days, 225 respectively. Hence, the current project duration is 235 days and ABDF is the critical path.

(b) What is the minimum amount of *extra* money you need to spend to reduce the project duration by 15 days? (5 marks)

First, we compute the cost slopes (we omit A as it is on every path and cannot be crashed):

Activity	Original Time	Original Cost	Crash Time	Crash Cost	Cost Slope
B ✓	90 days	\$10	80 days	\$70	\$6 per day
C	80 days	\$10	65 days	\$150	\$9.33 per day
D	80 days	\$10	70 days	\$40	\$3 per day
E	65 days	\$10	55 days	\$20	\$1 per day
F ✓	50 days	\$20	40 days	\$80	\$6 per day

A → B → D → F

\$6 (~~10 days~~ 5 days) \$3 (10 days) \$6 (10 days)

3 x 10 = 30

B or F

6 x 5 = 30

] – \$60

Other factors influencing Supply Chain Design

- Government Incentives and Policy
  - Tax Breaks
  - Tariff and non-tariff Barriers
  - Trade Agreements
- Legal
  - Intellectual Property Protection
  - Rule of Law
  - Employment Standards
- Network Effects
  - Innovation Clusters
  - Silicone Valley

The **lead time** is the time that it takes for goods to be shipped from the supplier.
The **reorder point** is the inventory level at which we place our order for the next cycle.
A **stock out** is the event when we do not have enough inventory to meet demand.

The safety stock (SS) corresponding to a reorder point ROP is any extra stock above the expected demand during lead time that we use to avoid stocking out during lead time. We compute safety stock as
SS = ROP – (Expected Demand During Lead Time) = ROP – E[DL].

The **(cycle) service level** corresponding to a reorder point ROP is the probability that we do not stock out during lead time if we place an order to the supplier when our at ROP. Given that we do not stock out during lead time precisely when DL ≤ ROP, the service level satisfies the equation
CSL = Pr(DL ≤ ROP).

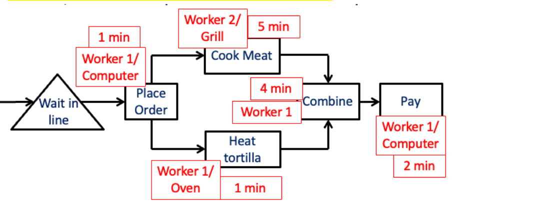
If we substitute $\rho = \lambda/\mu$, then we see that

$$\frac{\rho^2}{1-\rho} \times \frac{(C_A)^2 + (C_S)^2}{2} = \frac{\lambda^2}{\mu(\mu - \lambda)} \times \frac{(C_A)^2 + (C_S)^2}{2}.$$

Crashing Activities

1. Crash the "cheapest" critical activity first (the one with the lowest cost slope).
2. When crashing critical activities, pay attention to the fact that the critical path may change as we proceed.
3. New critical paths may emerge and we may now have to crash other activities.
4. If we want to reduce the process flow time, then we have to reduce the length of all critical paths simultaneously.

Utilization and Inventory Backlog



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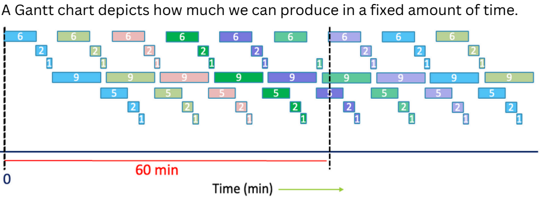
Gantt Charts: Process Flow

A Gantt chart depicts how much we can produce in a fixed amount of time.

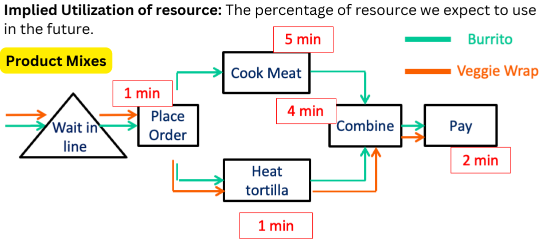


The capacity rate of a process might not equal the output rate.

Input Rate to the process: The rate at which units are entering the process.
Throughput rate of process: Actual output rate of the process.

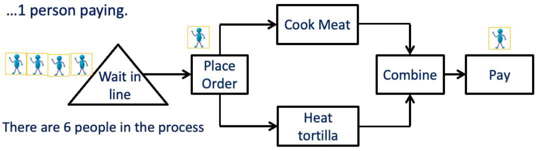
Utilization of a resource: The percentage of a resource actually being used.
Implied Utilization of resource: The percentage of resource we expect to use in the future.

Product Mixes



To analyze multiple flow units, we weight them to develop a **product mix.**
Inventory in the process: Number of flow units which are performing an activity or waiting in a buffer. (the units inside the process walls)

Suppose at 1:00 pm, our restaurant has...
...4 people waiting in line,
...1 person ordering, and
...1 person paying.



Each unit's individual flow time and the process inventory may depend on many things, ex. malfunction of resources, external factors. that's why we consider **averages.**

Little's Law

Little's law: In a stable process (long term average input equals throughput) the equation I = RT holds through.

Scheduling

Shortest Processing Time (SPT) Schedule: schedules tasks in order of process time (shortest time first). This schedule MINIMIZES the sum of completion times, or equivalently, the sum of weight times.

Weighted Shortest Processing Time (SPT) Schedule: schedules tasks based on the value of Time to Complete / Weight (smallest value first). This schedule minimizes the sum of weighted completion times.

Earliest Due Date (EDD) Schedule: schedules tasks in order of earliest due date.

Moore's algorithm: find a schedule by repeating these steps:
1. Find the EDD (earliest due date)
2. Find the first late task
3. Among all those tasks schedule up and including the first late in the EDD schedule, discard the longest one. This schedule minimizes the number of tasks completed after their deadline.

Relevant Formulas

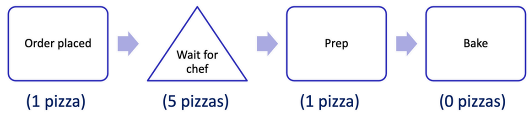
$$Capacity\ Rate\ of\ Resource = \frac{1}{Unit\ Load}$$

$$Resource\ Pool\ Capacity\ Rate = (capacity\ rate\ of\ each\ resource)\cdot(num.\ of\ resources\ in\ pool)$$

	Production Process	Service Process
Flow Unit	Materials	Customers
Input Rate	Raw material releasing rate	Customer arrival rate
Capacity Rate	Maximum output rate	Maximum service completion rate
Throughput Rate	Finished goods output rate	Customers departure rate (service completion rate)
Flow Time	Time required to turn materials into a product	Time that a customer is being served
Inventory	Amount of work-in-process	Number of customers being served

Inventory in a buffer counts all flow units in a buffer.
Inventory in an activity counts all flow units performing an activity.
Inventory in the process counts all flow units in the process (this includes all buffers and activities).

Example:



$$\textbf{Utilization (of a resource)} = \frac{\text{Throughput Rate (process)}}{\text{Capacity Rate (resource)}} \times 100\%$$

$$\textbf{Cycle time is time between outputs} = \frac{1}{\text{Throughput Rate}}$$

$$\textbf{Implied Utilization} = \frac{\text{Input Rate (process)}}{\text{Capacity Rate (resource)}}$$

Flow Time T = I/R

$$Cost\ Slope = \frac{(Crash\ Cost - Original\ Cost)}{(Original\ Time - Crash\ Time)}$$

$$Throughput\ Rate = \min(input\ rate, capacity\ rate)$$

$$Utilization\ of\ Resource = \frac{(Process\ Throughput\ Rate)}{(Resource\ Capacity\ Rate)}$$

$$Implied\ Utilization\ of\ Resource = \frac{(Process\ Input\ Rate)}{(Resource\ Capacity\ Rate)}$$

$$Q_{OPT} = \sqrt{\frac{2SD}{H}}$$

$$TC(Q) = \frac{Q}{2}H + \frac{D}{Q}S$$

$$\text{Cycle Time} = \frac{D}{Q_{OPT}}$$

I_q	Average inventory (in queue)
I_s	Average inventory (at the server)
$I=I_q+I_s$	Average inventory (in process)
T_q	Average waiting time (in queue)
T_s	Average service time (at the server)
$T=T_q+T_s$	Average flow time (in process)

Little's law

$$I = R \cdot T$$

Total Annual Cost = $TC(Q) = \frac{Q}{2}H + \frac{D}{Q}S$ = Annual holding Cost + Annual setup costs

$$Long\ Term\ Avg.\ Inventory\ \mathbf{I} = Long\ Term\ Avg.\ Throughput\ Rate\ \mathbf{R} \cdot Long\ Term\ Avg.\ Flow\ Time\ \mathbf{T}$$

$$WSPT\ schedule = schedule\ tasks\ with\ smallest\ \frac{(Time\ to\ Complete)}{Weight}\ first$$

$T_q = I_q/\lambda$: $I_q = \lambda T_q$

$I_s = \lambda T_s$

$I = \lambda T$

$T_s = \frac{1}{\mu}$

(utilisation rate)

$$\text{Average Cycle Stock} = \frac{\text{Order Quantity}}{2}$$

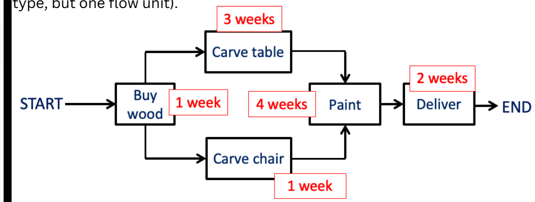
Process Management

Critical Path Analysis

Critical Path analysis helps us to find the **critical path** of the project, **crash costs and times,** and **how much money is needed to reduce the project by XX days**

Earned value analysis (BCWC, BCWS, ACWC) tells us if the project is **ahead of schedule, behind schedule, over budget** or **under budget)**

A project is a process that is often designed for a single flow unit (not just one type, but one flow unit).



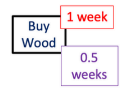
A project is a process that is often designed for a single flow unit (not just one type, but one flow unit).

Definition: A **critical path** in a project is a longest path, and its length is the expected project duration. **Critical activities** are activities on some critical path.

Paying Extra or Less to decrease Flow Time of Activity

Sometimes, we can decrease the flow time of an activity by paying extra to complete the task faster.

The crash time is 0.5 weeks, the crash cost is $750, and the cost slope is $500 per week.



The flow time for the "buy wood" activity is 1 week. Perhaps this activity costs $500.

If we pay extra for a faster shipment, the "buy wood" activity may only require 0.5 weeks. Perhaps this activity now costs $750

Definition: A **crash time of an activity** is a new possible theoretical flow time. The **crash cost** is the price to complete the activity at the crash time.

Definition: The **cost slope** is the extra cost we pay to shorten the flow time of an activity by one time unit.
This equals (Crash cost – Original cost)/(Original time – Crash time).
(750-500)/1-.05)

Delays

Definition: An **earned value analysis** can be used to determine if a project is over/under budget and ahead/behind schedule.

Definition: The budgeted cost of work scheduled to date for the activity is denoted by **BCWS**. This equals (% scheduled)*Budget.

*BCWS is how much we ***expected*** to spend based on the ***expected*** amount of completed work.

Definition: The budgeted cost of work completed to date for the activity is denoted by **BCWC**. This equals (% completed)*Budget.

*BCWC is how much we ***expected*** to spend based on the ***actual*** amount of completed work.

Definition: The actual cost of work completed to date for the activity is denoted by **ACWC**.

*ACWC is how much we ***actually*** spent based on the ***actual*** amount of completed work.

To compute the percent that an **activity** is ahead or behind schedule...

$$\frac{|\%\ Scheduled\ (activity) - \%\ Completed\ (activity)|}{\%\ Scheduled\ (activity)} \times 100\%$$

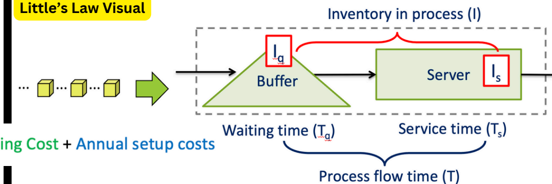
$$= \frac{|BCWS\ (activity) - BCWC\ (activity)|}{BCWS\ (activity)} \times 100\%$$

To compute the percent that the **project** is ahead or behind schedule...

$$\frac{|BCWS\ (Total) - BCWC\ (Total)|}{BCWS\ (Total)} \times 100\%$$

If BCWS<BCWC, we are ahead of schedule

Little's Law Visual



What might affect inventory in the buffer?
Utilization of the server ρ (higher ρ correlates with higher I_q).
The capacity rate μ of the server (higher μ correlates with lower I_q).
Variability in the arrivals and the server.

Number of orders per year	D/Q
Average (cycle) inventory	Q/2 if no safety stock otherwise Q/2 + SS

Annual Setup Cost	(D/Q) * S
Annual Holding Cost	(Q/2) * H or Total Cost / 2
Annual Total Cost	Annual Setup Cost + Annual Holding Cost