COMM 298 Formula Sheet

Bonds 1

Bond given Face Value (F), Coupon Rate (r), BEFORE YIELD TO MATURITY

$$P = I\left[\frac{1-(1+r)^{-n}}{r}\right]$$ + F / (1 + r)^n

- "After 5 years, want to sell. what is price of bond?"
- "What is price of bond?"

a) Find the price of a bond with a face value of $1,000 that pays 8% annual coupons and has 25 years until maturity. The current market rate of interest is 8% per year. r

PV of coupons

0 1 2 3 . . . . 24 25

80 80 80 80 80 1000

$$1000 = P_0 = 80\left[\frac{1-(1+0.08)^{-25}}{0.08}\right] + \frac{1000}{(1+0.08)^{25}}$$

e) Instead, suppose that after 5 years we sell the 8% coupon bond for $829.73 when the market rate of interest is 10%. *(recalculate the price)*

$$P_5 = 829.73 = 80\left[\frac{1-(1.10)^{-20}}{0.1}\right] + \frac{1000}{(1.1)^{20}}$$

original YTM = 25
sell after 5 years
P_5 = 80[(1-(1.10)^-20))/0.1] + 1000(1.1)^20
Yearly Coupon Rate = 80, 1000 * 0.10

P = price of bond
I = interest payments $(coupons)
r = market interest rate (can change over time)

F = face value of bond
n = periods until bond matures

KEY NOTES

- If Market Rate == Coupon Rate, BOND PRICE SAME.

Bond prices & market interest rates inversely related.

- *R goes DOWN -> bond price more valuable.*
  - *Selling at "Premium to Face Value"*
- *R goes UP -> bond price less valuable*
  - *Selling at "Discount to Face Value"*
- Coupon Rate in APR Terms
- Bond's Face Value is normally $1000

Perpetual Bonds

Is a stream of interest payments.

- Begins one period from now.
- Never matures

Price of Perpetual Bond: P = I / R

Bond Price AFTER YTM MATURES

$$P = I\left[\frac{1-(1+r)^{-n}}{r}\right] + F(1+r)^{-n}$$

pv of coupons   pv of original value

- *Price of a 30-year bond:*

P = $100[(1-(1.08)^-30)/0.08] + $1000(1.08)^-30
P = $1225.16

Bond Price ONE YEAR

P = F + I / (1 + r)

- *Price of a one-year bond:*

$$P = \frac{1000+80}{(1+0.08)} = 1018.52$$

$80 $1000

How much Bond Price Increase/Decrease?

Increase/Decrease = (New Price - Old Price) / Old Price

When *r* fell from 8% to 6%.

- The price of the perpetual bond increased by:
  (1666.67-1250)/1250 = 33.33%
  6% 8%
- The price of the 30-year bond increased by:
  (1550.59-1225.16)/1225.16 = 26.56%
  6% 8%
- The price of the one-year bond increased by:
  (1037.74-1018.52)/1018.52 = 1.88%
  6% 8%

Capital Gain

Capital Gain = Face Value - Purchase Price

- "How does bond buyer earn money from buying and holding until maturity?"

Q: How does the bond buyer earn money from buying our bond and holding it to maturity?
1. Coupons: $80 per year
2. Capital Gain: $1000 - 829.73 = 170.27 $1000
(Sales price or Face Value - Purchase price)
if "sold" at money - PP

Ex.
original value = $1000
holding to maturity = $829.73
$1000 - $829.73 = 170.27

Coupon Rate Calculation

$$I = F \times \frac{CR}{k}$$

CR = Coupon Rate (but static)
k = Number of coupon payments per year (often 2 payments a year)

- A Government of Canada bond has a face value of $1000, semi-annual coupon payments and a coupon rate of 6%. What is the coupon paid each period? $
- Solution:
  $F = \$1{,}000$
  $CR = 6\%$
  $k = 2$
  I = $1000 x 0.06/2 = $30 every 6 months

Bonds 2

Given YTM, calculate bond price. **BONDS ARE USUALLY SEMI-ANNUAL**

$YTM = r \times k$ market interest rate * amount of payments per year

- To find YTM for 6 months (semi-annual), divide YTM by 2.
  - Is used for semi-annual bond calculations

- An 8–year GOC bond with a $1,000 face value has semi-annual coupon payments, a 12% coupon rate, and a YTM of 10%. What is the bond valued at today? $P_0 = ?$
- Solution:
  1. Determine the $ amount of the coupons.
  2. Determine *r*, the effective periodic yield.
  3. Use the bond valuation formula.

Bonds in Canada   % PV

Issuer	Bond Type	Series	Coupon Rate	Coupon Freq.	Maturity	Price	Yield	Yield Type	Duration
ALBERTA	PROV		4.50	S	2040/12/01	124.19	3.08	M	15.86
ALBERTA	PROV		5.03	S	2018/12/17	109.29	1.28	M	2.34
ALBERTA	PROV		0.98	S	2016/05/27	100.00	0.36	M	0.00
ALBERTA	PROV	MTN	5.39	S	2013/06/14	100.00	0.00	M	0.00
ALBERTA	PROV		5.93	S	2016/09/16	101.46	0.54	M	0.27
ALTALINK LP	CORP		5.21	S	2016/12/16	101.80	1.72	M	0.51
ATLA CAP FIN AUTHORITY	PROV		4.45	S	2025/12/15	118.18	2.31	M	7.79
ATLA CAP FIN AUTHORITY	PROV		2.75	S	2016/10/28	100.00	3.64	M	5.38
BAIE-COMEAU	MUNI		4.15	S	2014/11/02	100.00	1.78	M	0.00
BAIE-COMEAU	MUNI		2.95	S	2014/12/02	100.00	1.88	M	0.00
BANK OF NOVA SCOTIA	CORP		4.99	S	2018/03/27	100.00	4.99	M	4.36
BANK OF NOVA SCOTIA	CORP	DPNT	2.74	S	2016/12/01	100.88	0.92	M	0.48
BANK OF NOVA SCOTIA	CORP		6.00	S	2018/10/03	100.00	6.00	M	4.25
BANK OF NOVA SCOTIA	CORP		4.15	S	2017/10/27	99.78	5.07	M	6.96
BAYTEX ENERGY	CORP		9.15	S	2016/08/26	109.25	7.11	M	4.25
BC FERRY SVCS	CORP		5.58	S	2038/01/11	131.07	3.52	M	13.50
BC MTN	PROV		8.50	S	2013/08/23	100.00	0.00	M	0.00
BC MUNICIPAL FINANCE	MUNI		4.88	S	2019/06/03	110.62	1.24	M	2.81
BC MUNICIPAL FINANCE	MUNI		4.60	S	2018/04/23	106.67	1.00	M	1.80
BCIMC REALTY CORP	CORP		5.65	S	2018/01/05	106.73	1.32	M	1.49
BELL CANADA	CORP		8.88	S	2026/04/17	136.97	4.25	M	7.05

http://www.globeinvestor.com/servlet/Page/document/v5/data/bonds/

1. $1000 * 0.12/2 = 60 (coupon amount)
2. We know that YTM = 0.10, so semi annual is 0.05
3. We know that:

n = 8, multiply by 2 because semi-annual
F = 1000
I = 60 (interest rate)

$$P = \$60[(1-(1.05)^{-16})/0.05] + \$1000(1.05)^{-16}$$
$$P = \$1108.38$$

(use bond after maturity formula)

Don't confuse coupon rate with yield to maturity!
$ coupon   r

- **Coupon rate:** remains unchanged over the life of the bond. It determines the $ coupon payments.
- **YTM:** changes because market interest rates change and the bond's risk can changed, which cause the bond price to change.

The YTM measures the average annual (APR) rate of return on the bond you would earn if you were to buy the bond today and hold it until maturity.

- Coupon Rate STAYS Static
- YTM changes depending on the interest rate
- YTM - buying the bond today and holding until maturity
- Coupon Rate- buying back then

- **Default premium:** difference between the YTM on a risky bond and a government bond of similar maturity and coupon.
- Bond rating firms rate the default risk of risky bonds. High investment grades are in the range from BBB to AAA. **Junk (or speculative) bonds** are rated below BBB.

Calculate YTM (**WILL ASK FOR SETUP ON EXAM**)

The solution technique to use is:

1. Enter the bond's price on the left side of the bond pricing equation and enter all your inputs on the right hand side.
2. Use trial and error until you find the correct *r* that makes the LHS = RHS.
3. Use *YTM* = *r* × *k* to obtain the YTM.

If…	then…	and the bond trades at
YTM < CR	P > F	a **premium**
YTM = CR	P = F	**par** FV
YTM > CR	P < F	a **discount**

1. Enter bond price on left side of bond equation
2. Enter inputs on the right side,
   a. Note that IS SEMI ANNUAL, so n = 4 * 2 = **8**
3. TRY TO FIND INTEREST RATE
4. Use the YTM = r x k formula

YTM less then coupon? People would buy coupon for more, so at premium.

YTM is more then the coupon? People rather buy coupon, so bond sells at cheaper price.

A 4-year $1,000 bond with a coupon rate of 6%, paying coupons semi-annually, is priced at $1035.85. We wish to determine its YTM. First: set up the equation to determine *r*.

$$\$1035.85 = \$30[(1-(1+r_{sa})^{-8})/r_{sa}] + \$1000(1+r_{sa})^{-8}$$

(use bond after maturity formula)

Current Yield

Current Yield = Annual Coupon / Current Price

- Example:
  A bond with a $1,000 face value has an 8% coupon rate and is trading at $953.80. $80
  Current Yield = $80 / $953.80 = 8.39%

KEY NOTES

- **Bid Price**- price that a bond dealer is prepared to bid
- **Ask Price**- price that bond dealer is willing to sell at
- YTM changes depending on market interest rate
- Bond Prices are quoted as a percent of face value
- bonds with higher default risk have higher YTM
  - bond trading at $1045.90 is 104.59 (% amount)
- Coupon rate static

Bonds 3

Vocabulary

- Expected Rate of Return- Return that an investor wants for investment of certain risk
- Realized Rate of Return- What they actually get when they sell the bond. is **ANNUAL** realized rate of return.

Bond Yields

- Bond Yields affected by
  - Real rate of interest
  - Premium for expected future inflation
  - An interest rate risk premium
  - Default risk premium
  - Liquidity premium

Realized Rate of Return

$$ROR = \left[\frac{(\text{selling price} + \text{FVRC})}{\text{purchase price}}\right]^{\frac{1}{n}} - 1$$

YTM assumes that:

1. You hold the bond until maturity.
2. You reinvest the coupons at that YTM.

FVRC = Future Value of Reinvested Coupons

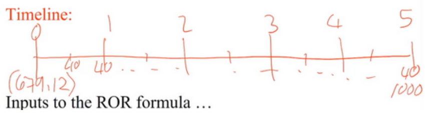
Selling Price and Purchase Price of bond, is part of capital gain

Realized Rate of Return WITH MATURITY

- You pay $679.12 to buy a 5-year, $1,000 face, 8% coupon, semi-annual payments bond with a YTM of 18%.
- *You hold the bond until maturity*, and you reinvest the coupons in a savings account that pays 4% per year, compounded semi-annually.
- **What is your ROR?**

Selling price = $1000, since it's until maturity

- Timeline:



- Inputs to the ROR formula …

TIP. ALWAYS DRAW A TIMELINE

- Need to convert the market interest rate to semi-annual
  - R = 0.04 / 2 = 0.02
- Coupon Rate stays the same (0.04 * 1000) = 20

What is the future value of the reinvested coupons?

FV of Annuity inputs: 0x10=400
amount of each coupon = $40
number of coupons you receive = 10

FVRC = CR[((1 + r)^n - 1))/r]

FVRC =$40[((1.02)^{10}-1)/0.02] = 437.99

- Using the realized rate of return formula…

$$ROR = \left[\frac{(\$1{,}000 + \$437.99)}{\$679.12}\right]^{\frac{1}{5}} - 1 = .161882$$

16.19%pa

- 5 is from "5 year", and the rest is all compiled together from beginning
- *This is less than your expected YTM when you bought the bond. Why?* 16.19%pa 18%
- Did not re-invest coupons at original YTM. Reinvested at 4%.

Realized Rate of Return BEFORE MATURITY. NEED **SALES PRICE**

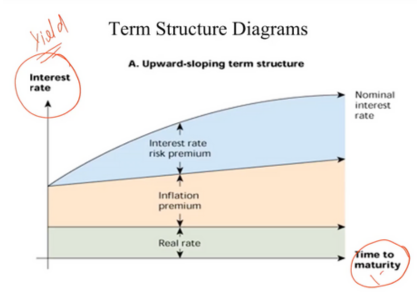
- What if you sold the bond before maturity?
  - Then use the actual selling price iso face value of $1,000 (also adjust *n* to reflect your holding period!)

Term Structures

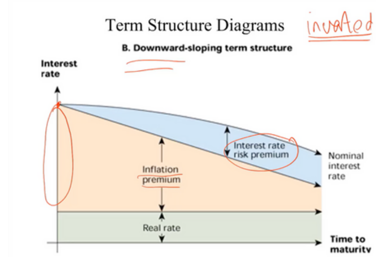
Basic components of bond yields

	5 year XXX bond
Real interest rate	1%
Inflation rate	2%
Interest rate risk	0.5%
Nominal interest rate	3.5% => 'market rate'
Other:	
Default/credit risk	2%
Liquidity	1%
Total bond yield:	6.5%

Yield



inverted



- **Nominal returns:** returns measured in dollar terms.
- **Real returns:** returns measured in terms of purchasing power.

The **Fisher Equation** relates the real rate of return to the nominal rate of return and the rate of inflation:

$$1 + \text{real rate} = \frac{1 + \text{nominal rate}}{1 + \text{inflation rate}}$$

You can rewrite this approximately as:

real rate = nominal rate – inflation rate

Holding Period of Return

- Using the realized rate of return formula…

$$ROR = \left[\frac{(\$1{,}000 + \$437.99)}{\$679.12}\right]^{\frac{1}{5}} - 1 = .161882$$

- *Can you adjust the above to calculate your HPR over 5 years?*

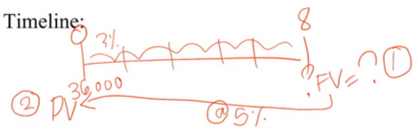
$$HPR_{5y} = \left[\frac{(\$1{,}000 + \$437.99)}{\$679.12}\right]^{\frac{5}{5}} - 1 = 1.11743 = \boxed{111.74\%}$$

Inflation Example 1

You want to do an MBA 8 years from now. MBA tuition is currently at \$36,000 per year and is expected to increase by 3% per year. If you earn 5% per year on investments, how much must you invest today to pay the first year's tuition?

inflation

Timeline:



Inflation Example 1

Step 1: Calculate FV of \$36,000 at 3% inflation to see what the first year's tuition will be:

$FV = \$36{,}000(1.03)^8 = \$45{,}603.72$ Nominal CF

Step 2: Calculate the PV of the above at a 5% discount rate (the rate of return that your investments is estimated to earn)

$PV = 45{,}603.72/(1.05)^8 = \$30{,}866.39$

OR:

Step 1: Calculate the real rate: $1.05/1.03 - 1 = 0.01941748$

Step 2: Discount real cash flow at real discount rate:

$36{,}000/(1+0.01941748)^8 = \$$ 30 866.39

Important Rule

Discount **nominal** cash flows using the **nominal** interest rate

and

Discount **real** cash flows using the **real** interest rate.

Real or Nominal?

- **Most analysis we will do will assume nominal rates and will discount nominal cash flows. (Assume this unless otherwise indicated!)**
- When one set of cash flows is presented in real terms, then other nominal cash flows and rates must be adjusted in order to compare, add or subtract the cash flows.
- As noted earlier, do not mix nominal and real or you will have garbage results!

Providing for Retirement

- Expected inflation is a significant variable in retirement planning, tuition savings plans, choice of vocation, or any long term financial planning. Even a low rate of inflation can have a major negative effect on people who will receive relatively fixed nominal income or returns.
- From the Fisher equation, we see that with high inflation, the realized real rate may be negative!

Stock Valuation I

Debt
- Need to pay interest
- Cost of doing interest
- Tax deductible

Equity
- Ownership of company (stock)
- All equity firm cannot go bankrupt

Dividends
- residual cash flows of a firm
- board and management decide how much to give to shareholders

Shareholders
- Shareholders own a company
- After all obligations have been paid, shareholders get to decide the residual cashflow amongst themselves

Preferred Shares
- Pay a **fixed periodic dividend**
- Is usually non-voting
- Have priority over common shares
- Is a perpetuity

Common Share Valuation
- Lifetime of an equity investment is potentially infinite
- Cashflows on shares are not guaranteed, are uncertain
- No easy way to observe return that investors requite on common shares

Types of Dividend Payments
1. Zero Growth
2. Constant Growth
3. Non-constant Growth

Stock Price Valuation

$$P_0 = Div_1(1+r)^{-1} + Div_2(1+r)^{-2} + Div_3(1+r)^{-3} + \ldots$$

Stock price is the present value of all expected future dividends.
Expression for the stock price is called the **dividend discount formula**.

Zero Growth Calculations

① **Zero Growth Dividend Model**

Zero growth implies that:

$DIV_1 = DIV_2 = \cdots = D$, a constant

\$10 = \$10

Since the cash flow is always the same, the PV is that for a perpetuity:

$$P_0 = \frac{D}{r}$$

Gives the PV one period before the cash flows start

Example 1

Bankcity's perpetual preferred stock pays a \$2 dividend and is priced to yield 13 percent. What is the price of this issue according to the zero growth model?

$$\frac{D}{r} = \frac{2}{0.13} = 15.38 = P_0$$

Constant Growth Calculations

TVM

Growing Perpetuities

- A growing perpetuity is characterized by an **infinite** series of **periodic** payments, **growing** at a **constant rate** each period. e.g. 5%
- The present value of a growing perpetuity:

$$PV = \frac{A}{r-g}$$ Gives the PV one period before the cash flows start

where:
A is the amount of the first payment → D_1
This is often referred to as the **Gordon Growth Model**
Key assumption: r>g

Constant-Growth Dividend Discount Model

The series of dividends for a share of stock can be considered a growing perpetuity if the dividend payment increases at the same rate each period. g

$D_0(1+g)$

$D_1 = D_0(1+g)$ $P_0 = \frac{D_1}{r-g}$

Constant growth dividend example

A share of stock just paid an annual dividend of $2 per share. Dividends are expected to increase at the rate of 5% per year forever. Future dividends are expected to be: g

$D_0 = \$2$
$D_1 = \$2(1+0.05) = \$2.10$
$D_2 = \$2(1+0.05)^2 = \$2.21$
$D_t = \$2(1+0.05)^t$

Assuming dividends grow at a constant rate, **the constant-growth dividend model** is:

$$P_0 = \frac{DIV_1}{r-g} = \frac{DIV_0(1+g)}{r-g}$$

where next period's dividend establishes the starting point, r is the required rate of return, and g is the estimated dividend growth rate.

Note: r must be > g!

Rearranging to solve for r as an expected return,we obtain:

$$r = \frac{DIV_1}{P_0} + g$$ →dividend yield

We see that the expected stock return consists of the dividend yield plus the expected future growth in dividends.

Example

- **Note that you need to differentiate between D_0 and D_1**

Valuation example: constant dividend growth

A stock pays annual dividends
$DIV_0 = \$1.50$
$g = 4\%$ per year
$r = 8.5\%$.

a) What is the stock price?
b) What if r rises to 10%?
c) What if g rises to 5%? (assume $r = 8.5\%$)

- **higher discount rate -> lower stock price**
- **higher growth -> higher stock price**

Gordon Growth Calculations (D_0 is not included in P_0

Gordon Growth Model – Ex vs Cum dividend

- Our **constant growth** model: $P_0 = \frac{DIV_1}{r-g} = \frac{DIV_0(1+g)}{r-g}$
- This is the price of a share that has **just paid a dividend D₀**. ex
- It is also called the **'ex dividend price'** and the more general formula (written in terms of D_t) becomes:

$$P_t^{ex} = \frac{D_{t+1}}{r-g} = \frac{D_t(1+g)}{r-g}$$

Gordon Growth Model – Ex vs Cum dividend

- **If the dividend has not been paid**, the share is trading **'cum dividend'** and the **dividend (D_t)** is simply **added to the ex-dividend price** we calculated.
- A firm's ***cum-dividend*** value is therefore:

$$PV_t^{cum} = \boldsymbol{D_t} + PV_t^{ex} = \boldsymbol{D_t} + \frac{D_t(1+g)}{r-g}$$

which can be reduced to:

$$= D_t\left(1 + \frac{1+g}{r-g}\right) = D_t\left(\frac{r-g+1+g}{r-g}\right)$$

$$\boldsymbol{PV_t^{cum} = D_t \frac{1+r}{r-g}}$$

Gordon Growth Model - Summary

- A firm's ***ex-dividend*** stock price is:

$$P_t^{ex} = \frac{D_{t+1}}{r-g} = \frac{D_t(1+g)}{r-g}$$

The next dividend is D_{t+1} ; the most recent dividend is D_t (it has already been paid and thus is excluded from P^{ex}).

- A firm's ***cum-dividend*** stock price is:

$$P_t^{cum} = \frac{D_t(1+r)}{r-g}$$

$P^{cum} > P^{ex}$ because it includes an additional dividend at t.

Gordon Growth Model - Summary

Formula assumes r>g, ***but what happens if r<g***?

$\frac{D_0(1+g)}{r-g}$

- If the required rate of return is less than the growth rate of dividends per share, the result is a negative value, rendering the model worthless.
- If the required rate of return is the same as the growth rate, the value per share approaches infinity.

Valuation example: negative dividend growth

Can g be negative?

If a company's dividends are expected to **decline** at a constant rate, then g will be negative.

Suppose dividends on a stock today are $1.20 per share and dividends are expected to decrease each year at a rate of 2% per year, forever. If the required rate of return is 8%, what is the value of a share of stock?

Valuation example: negative dividend growth

$$P_0 = \frac{D_0(1+g)}{r-g}$$
$$= \frac{1.2(1-0.02)}{0.08-(-0.02)}$$
$$= \frac{1.176}{0.1}$$
$$= 11.76$$

Zero growth:
$$\frac{1.2}{0.08} = 15$$

Supernormal Growth

Non-Constant Growth Model

Some firms' dividends grow at supernormal growth rates over a finite length of time. The growth rate cannot exceed the required return indefinitely, but it certainly could do so for a number of years.

Supernormal growth example 1:

D x (1+r)^n

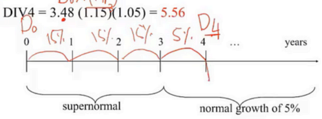
DIV1 = 3.48 (1.15) = 4

DIV2 = 3.48 $(1.15)^2$ = 4.60

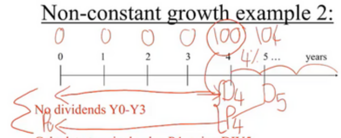
DIV3 = 3.48 $(1.15)^3$ = 5.29

D0 x (1+r)^3

DIV4 = 3.48 $(1.15)^3$(1.05) = 5.56



Non-constant growth example 2:



No dividends Y0-Y3

Calculate terminal value P4, using DIV5:

DIV5 = 100*(1.04) = 104

P4= $\boxed{\text{DIV5/(r-g)}}$ + DIV4

=104/(.1-.04) + 100

= $1,833.33

P0 = 1,833.33/$(1.1)^4$

= 1,252.19

Price Earnings Ratio

Price-Earnings Ratio P/E

- **Price-Earnings Ratio**: the ratio of price per share to earnings per share. Financial analysts often rely on price-earnings (P/E) ratios.

$$P/E = \frac{\text{Price}}{\text{EPS}}$$

- Earnings per share (EPS): net income divided by the current number of shares outstanding.

Example:
Stock valuation using P/E multiples

Values for subject firm	
Consensus EPS forecast	$4.50
Current stock price	$28.00
Values for peer group	
Median P/E	9.00

Use the P/E multiples method to determine if the subject firm is over or under valued relative to its peer group…

Supernormal Growth

E(r) Expected vs. Realized Return ROR

- An investor's average annualized **realized** rate of return from holding a stock for a given number of years, n, is:

$$ROR = \left(\frac{\text{Selling price} + \text{FV of reinvested dividends}}{\text{Purchase price}}\right)^{\frac{1}{n}} - 1$$

- Note that the numerator reflects **realized** cash flows.

Realized return example

You buy a share of a company for $22, hold the stock for 3 years, and receive a total of three annual dividends at the end of each year, each in the amount of $1. You invest these dividends in a bank account which pays an effective annual interest rate of 5%. At the end of the 3 years, you sell the stock for $27. What is your ROR?

Realized return example

- Purchase price: $22
- Sales price: $27
- Holding period: 3 years

FVRD = $\boxed{\$1[((1.05)^3 - 1)/0.05]}$

=$3.15

ROR = $[(27 + 3.15)/22]^{1/3}$-1

= 0.1108 = 11.08%