

Normal Forms

Functional dependency: one attribute determines another through a functional dependency.

Ex. If **Department** determines Address but not Name, we say that there's a functional dependency from Department to Address. But Department is NOT a key.

X determines Y.

if $t1.X = t2.X$ then $t1.Y = t2.Y$

It is a statement about all allowable instances.

- Must be identified by application semantics
- Given some instance $r1$ of R, we can check if $r1$ violates some FD f , but cannot tell if f holds over R.

Examples of Functional Dependencies

Trivial case is where:

FD: PostalCode $\rightarrow$ Province

So PostalCode, House # $\rightarrow$ Postal Code

Given some FDs, we can often infer additional FDs:

Reflexivity: If $Y \subseteq X$, then $X \rightarrow Y$

e.g., city, major $\rightarrow$ city

Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z

e.g., if sid $\rightarrow$ city, then sid, major $\rightarrow$ city, major

Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

sid $\rightarrow$ city, city $\rightarrow$ areacode implies sid $\rightarrow$ areacode

Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$

e.g., if sid $\rightarrow$ areacode and sid $\rightarrow$ city, then sid $\rightarrow$ areacode, city

Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$

e.g., if sid $\rightarrow$ areacode, city then sid $\rightarrow$ areacode, and sid $\rightarrow$ city

A couple of additional rules (that follow from axioms):

Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$

e.g., if sid $\rightarrow$ areacode and sid $\rightarrow$ city, then sid $\rightarrow$ areacode, city

Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$

e.g., if sid $\rightarrow$ areacode, city then sid $\rightarrow$ areacode, and sid $\rightarrow$ city

Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$

e.g., if sid $\rightarrow$ areacode and sid $\rightarrow$ city, then sid $\rightarrow$ areacode, city

- $X \rightarrow Y$ Given
- $X \rightarrow Z$ Given
- $X \rightarrow XY$ 1, augmentation
- $XY \rightarrow ZY$ 2, augmentation
- $X \rightarrow ZY$ 3, 4, transitivity

Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$

e.g., if sid $\rightarrow$ areacode, city then sid $\rightarrow$ areacode, and sid $\rightarrow$ city

- $X \rightarrow YZ$ Given
- $YZ \rightarrow Y$ Reflexivity
- $YZ \rightarrow Z$ Reflexivity
- $X \rightarrow Y$ 1, 2, transitivity
- $X \rightarrow Z$ 1, 3, transitivity

Show that (sname, p#) is a superkey of SupplierPart(sname, city, status, p#, pname, qty)

Proof has two parts:

a. Show: (sname, p#) is a (super)key

- sname, p# $\rightarrow$ sname, p#
- sname $\rightarrow$ city
- sname $\rightarrow$ status
- sname, p# $\rightarrow$ city, p#
- sname, p# $\rightarrow$ status, p#
- sname, p# $\rightarrow$ sname, p#, status
- sname, p# $\rightarrow$ sname, p#, status, city
- sname, p# $\rightarrow$ sname, p#, status, city, qty
- sname, p# $\rightarrow$ sname, p#, status, city, qty, pname

b. Show: (sname, p#) is a minimal key of SupplierPart(sname, city, status, p#, pname, qty)

- p# does not appear on the RHS of any FD therefore except for p# itself, nothing else determines p#
- specifically, sname $\rightarrow$ p# does not hold
- therefore, sname is not a key
- similarly, p# is not a key

Specifically, for combinations of sname and p#:

- sname, p# $\rightarrow$ sname, p#, city, status, pname, qty
- sname $\rightarrow$ sname, city, status
- p# $\rightarrow$ p#, pname

- Scared you're going to mess up? **Closure** is a fool-proof method of checking FDs and finding implicit FDs.

- Closure for a set of attributes X is denoted X^+

- X^+ includes all attributes of the relation IFF X is a (super)key

- Algorithm for finding Closure of X:

Let Closure = X

Until Closure doesn't change do

if $a_1, \dots, a_n \rightarrow C$ is a FD and $\{a_1, \dots, a_n\} \subseteq \text{Closure}$

Then add C to Closure

SupplierPart(sname, city, status, p#, pname, qty)

Ex: sname, p# $\rightarrow$ {sname, p#, city, status, pname, qty} $\leftarrow$ superkey

sname $\rightarrow$ {sname, city, status}

p# $\rightarrow$ {pname}

So seeing if a set of attributes is a key means checking to see if it's closure is all the attributes - pretty simple

Normalization

Normalization is the process of removing redundancy from data

Four important normal forms:

- First Normal Form (1NF)
- Second Normal Form (2NF)
- Boyce-Codd Normal Form (BCNF)
- Third Normal Form (3NF)
- If a relation is in a certain normal form, certain problems are avoided/minimized
- Normal forms can help decide whether decomposition (splitting tables) will help

1NF

Each attribute in a tuple has only one value (can't be an array).

E.g., for "postal code" you can't have both V6T 1Z4 and V6S 1W6

Why do we need it? Codd's original version allowed multi-valued attributes

Client ID	Postal Code
1	V6T 1Z4, V6S 1W6

Normalize to 1NF

Client ID	Postal Code
1	V6T 1Z4
1	V6S 1W6

Can't have multiple values in a single attribute

2NF

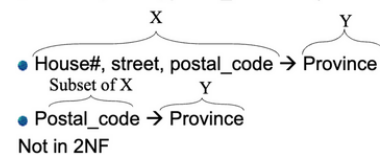
There are no partial key dependencies.

A relation is in 2NF if it is in 1NF and for every FD, $X \rightarrow Y$ where X is a (minimal) key and Y is a non-key attribute, then no proper subset of X determines Y.

Ex. This address relation is not in 2NF

e.g., the address relation is not in 2NF:

- House#, street, postal_code is a (minimal) key



Not in 2NF

2NF

There are no partial key dependencies.

A relation is in 2NF if it is in 1NF and for every FD, $X \rightarrow Y$ where X is a (minimal) key and Y is a non-key attribute, then no proper subset of X determines Y.

Ex. This address relation is not in 2NF

Boyce-Codd Normal Form (BCNF)

A relation R is in BCNF if $X \rightarrow Y$ is a non-trivial dependency in R, then X is a superkey for R.

Ex. Whenever a set of attributes R determine another attribute, it should determine all the attributes of R.

Check if all the left parts of the FD are a superkey. If they are, then it is in BCNF, and if not, they need to be decomposed.

Decomposition

Decomposition of R replaces R by two or more relations.

- Each new relation contains a subset of the attributes of R (and no attributes not appearing in R)
- Every attribute of R appears in at least one new relation.

Definition: $R_1 \bowtie R_2$ is the (natural) join of the two relations; i.e., each tuple of R_1 is concatenated with every tuple in R_2 having the same values on the common attributes.

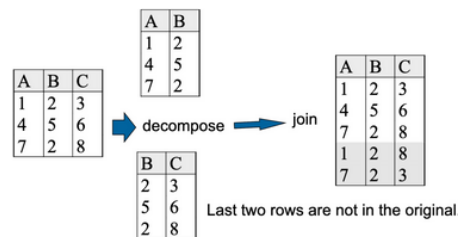
R_1		R_2		$R_1 \bowtie R_2$																															
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Lossless-Join Decomposition

Informally: If we break a relation, R, into bits, when we put the bits back together, we should get exactly R back again

Formally: Decomposition of R into X and Y is lossless-join w.r.t. a set of FDs F if, for every instance r that satisfies F:

- If we JOIN the X-part of r with the Y-part of r the result is exactly R
- It is *always* true that r is a subset of the JOIN of its X-part and Y-part, even if the join isn't lossless
- In general, the other direction does not hold - you may get back additional information! If it does hold, the decomposition is a lossless-join.
- Note: The word loss in lossless refers to loss of information, not to loss of tuples. In fact, for "loss of information" a better way to refer to it might be "addition of spurious information".



Finding Minimal Keys

Ex. Find all minimal keys for R(ABCD) where FDs are $AB \rightarrow C$ and $C \rightarrow BD$.

- List only attributes that appear on the RHS of the FDs (only what can be derived). Put on the right.

Left	Middle	Right
		D

- List all the attributes that only ever appear on the LHS of an FD (or do not appear in the FD at all, ex. things that have to be a part of any key). Put on the left.

Left	Middle	Right
A		D

- List attributes that appear on the LHS and the RHS of the FDs (not obviously required and may help you derive). Put in middle

Left	Middle	Right
A	BC	D

- Take the closure of the attributes in the left column.

$\{A\}^+ = \{A\}$

Are all of the attributes there? If so, you have found a minimal key. If not, start adding in attributes from the middle column to see if you can determine all the attributes of the relation.

Examples of 3NF

A Relation R is in 3NF if:

If $X \rightarrow b$ is a non-trivial dependency in R, then X is a superkey for R or **B is part of a (minimal) key**.

Note: b must be part of a key not part of a superkey (if a key exists, all attributes are part of a superkey)

Example:

R(Unit, Company, Product)

Unit $\rightarrow$ Company

Company, Product $\rightarrow$ Unit

Keys: {Company, Product}, {Unit, Product}

Rule: for all non-trivial functional dependencies in a relation R of the form $X \rightarrow b$, it must be the case that X is a superkey of R or **b is part of a key**.

Minimal Cover G for a set of FDs F:

- Closure of F = closure of G (i.e., imply the same FDs)
- Right hand side of each FD in G is a single attribute
- If we delete an FD in G or delete attributes from an FD in G, the closure changes

e.g., $A \rightarrow B, ABCD \rightarrow E, EF \rightarrow GH, ACDF \rightarrow EG$ has the following minimal cover:

- $A \rightarrow B, ACD \rightarrow E, EF \rightarrow G$ and $EF \rightarrow H$

- Put FDs in standard form (have only one attribute on RHS)
- Minimize LHS of each FD
- Delete Redundant FDs

Example:

$A \rightarrow B, ABCD \rightarrow E, EF \rightarrow G, EF \rightarrow H, ACDF \rightarrow EG$

- Replace last rule with
 - $ACDF \rightarrow E$
 - $ACDF \rightarrow G$

- Put FDs in standard form (have only one attribute on RHS)
- Minimize LHS of each FD
- Delete Redundant FDs

Example:

$A \rightarrow B, ABCD \rightarrow E, EF \rightarrow G, EF \rightarrow H, ACDF \rightarrow E, ACDF \rightarrow G$

- Can we take anything away from the LHS?
 - $ACD^+ = ABCDE$, (crucially includes B) so remove B from the FD

- Put FDs in standard form (have only one attribute on RHS)
- Minimize LHS of each FD
- Delete Redundant FDs

Example:

$A \rightarrow B, ACD \rightarrow E, EF \rightarrow G, EF \rightarrow H, ACDF \rightarrow E, ACDF \rightarrow G$

- Let's find $ACDF^+$ *without* considering the highlighted FDs
 - $ACDF^+ = ACDFEBGH$, so I can remove the highlighted rules

- Final answer: $A \rightarrow B, ACD \rightarrow E, EF \rightarrow G, EF \rightarrow H$

Examples of BCNF

Remember BCNF def: For all non-trivial functional dependencies $X \rightarrow b$, X must be a superkey for a relation to be in BCNF

Relation: R(ABCD) FD: $B \rightarrow C, D \rightarrow A$

Keys?

$A^+ = \{A\}$

$B^+ = \{B, C\}$

$C^+ = \{C\}$

$D^+ = \{A, D\}$

$BD^+ = \{B, D, C, A\}$

BD is the only key

Look at FD $B \rightarrow C$. Is B a superkey?

No. Decompose

$R1(B, C), R2(A, B, D)$

Look at FD $D \rightarrow A$. Is D a superkey?

No. Decompose

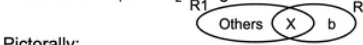
$R3(D, A), R4(D, B)$

Final answer: $R1(B, C), R3(D, A), R4(D, B)$

Let R be a relation with attributes A, and FD be a set of FDs on R s.t. all FDs determine a single attribute

Pick any $f \in$ FD that violates BCNF of the form $X \rightarrow b$
Decompose R into two relations: $R_1(A-b)$ & $R_2(X \cup b)$

Recurse on R_1 and R_2 using FD



Pictorially:

